D&F;3.4: 2, 7, 17*

D&F;4.1: 4, 7*

D&F;4.2: 8, 10

As in problem 3.1:41, we define the derived subgroup of $G$ to be the group $G' = \langle x^{-1}y^{-1}xy \mid x, y \in G \rangle$. We define the derived series of $G$ to be the series of subgroups defined by

$$G^{(0)} = G, \quad G^{(i+1)} = (G^{(i)})'.$$

(i.e. $G^{(1)} = G'$, $G^{(2)} = G''$, etc.)

1) Let $G$ be a finite group. Prove, that the following statements are equivalent:

a) $G$ is solvable

b) There is a composition series

$$\langle 1 \rangle = N_0 \leq N_1 \leq \cdots \leq N_{l-1} \leq N_l = G$$

with $N_i/N_{i-1}$ cyclic of prime order.

c) For every normal subgroup $N \triangleleft G$, both $N$ and $G/N$ are solvable

d) The derived series of $G$ reaches the trivial subgroup, i.e. there is an index $i$, such that $G^{(i)} = \langle 1_G \rangle$.

(Problems marked with a * are bonus problems.)