1) Let $G$ be a group. $G$ acts on itself by conjugation, that is by

$$(h, g) \mapsto h^g := g^{-1} g h$$

Show that this satisfies the axioms of a group action.

In GAP, you can construct the homomorphism induced by a group action in the following way:

\[
\text{ActionHomomorphism}(G, A, \text{actionfun});
\]

where $G$ is a group, $A$ is the domain and \text{actionfun} indicates the action. The Image of such a homomorphism is the resulting permutation group, Kernel is the kernel of the action. Predefined actions (possible values for \text{actionfun}) are:

- \text{OnPoints} Permutation group on points, Matrix group on vectors, Group on its elements or subgroups by conjugation.
- \text{OnRight} Right multiplication (group on its elements)
- \text{OnTuples} A permutation group on tuples of points
- \text{OnSets} A permutation group on sets of points (the sets must be sorted)

For example

\[
g := \text{SymmetricGroup}(3);
\]
\[
\text{ActionHomomorphism}(g, \text{AsList}(g), \text{OnRight});
\]
\[
\text{ActionHomomorphism}(g, [[1, 2], [1, 3], [2, 3]], \text{OnSets});
\]

gives the action of $S_3$ on itself by right multiplication, respectively on sets of 2 points.

In the same syntax, you can specify:

\[
\text{Stabilizer}(g, \text{point}, \text{actionfun});
\]

There also are \text{Centralizer}(G, \text{elm}), \text{Centre}(G) and \text{Normalizer}(G, H).

2) (GAP) Let $G = S_5$ (\text{SymmetricGroup}(5)). Compute the centralizers for one element of each possible cycle type in $G$. 
3) (GAP) Take the elements of $S_5$ that are 5-cycles

\[
\text{Filtered(AsList(g),i->CycleStructurePerm(i)=[,,1]);}
\]

and form the different subgroups generated by them.
Let $S_5$ act on the set $A$ of these subgroups by conjugation (OnPoints). What is the image of this action? Is the action faithful?
Deduce that $S_5$ is isomorphic to two different subgroups of $S_6$.

4) (GAP) Take the group of rotations and reflections of a cube (Problem 3 on the last homework) and let it act on the diagonals of the cube (represent them as sets of points). What image group do you get?
Describe the elements of the kernel of this action.