Let $R = \mathbb{Z}[i] = \{a + bi \in \mathbb{C} \mid a, b \in \mathbb{Z}\}$ the ring of Gaussian Integers. For $r = a + bi \in R$ let $N(r) = a^2 + b^2$. Show that $R$ is an euclidean ring with this norm. Determine a gcd of $7 + i$ and $-1 + 5i$.

2) Give an example of a ring which is not noetherian (i.e. which possesses infinite ascending chains of ideals).

Review: Ideal, prime ideal, maximal ideal, prime element, irreducible element, zero divisor, unit, integral domain, principal ideal domain, euclidean ring, unique factorization domain, noetherian ring. Which properties imply which ones? Give (counter)examples to show that the remaining possible implications are wrong.