40) a) Let \( a, b \) be elements of a poset \( P \). Prove that
\[
\mu(a, b) = \sum_{i \geq 0} (-1)^i c_i,
\]
where \( c_i \) is the number of chains \( a = x_0 < x_1 < \cdots < x_i = b \). (Hint: Show that the right hand side fulfills the defining property.)
b) For a poset \( P \), let \( P^* \) be the poset obtained by reversing the order relation (that is \( x \leq_P y \) iff \( y \leq_P x \)). Conclude that \( \mu_{P^*}(a, b) = \mu_P(b, a) \).

40a) (For those who have seen algebraic topology). For a poset \( P \) with a global minimal element \( 0 \) and a global maximal element \( 1 \), we define a simplicial complex \( \Delta(P) \), called the order complex, as follows: The elements of \( P \) are the vertices of \( \Delta(P) \), the chains of \( P \) are the faces. Show that \( \mu_P(0, 1) = \chi(\Delta(P)) - 1 \), where \( \chi \) is the ordinary Euler characteristic.

41) Determine \( \mu(0, 1) = \mu((1), D_8) \) for the lattice of subgroups of \( D_8 \), the dihedral group of order 8, given on the side.

42) a) Let \( X \) be a set and assume \( S \) is partitioned in two different ways into \( m \) cells:
\[
X = A_1 \cup A_2 \cup \cdots \cup A_m = B_1 \cup \cdots \cup B_m,
\]
that is \( A_i \cap A_j = \emptyset = B_i \cap B_j \) if \( i \neq j \). Assume that any \( k \) of the \( A_i \) intersect at least \( k \) of the \( B_j \). Show that it is possible to find a simultaneous set of \( m \) representatives for the two partitions.
b) Let \( G \) be a finite group and \( S \leq G \) a subgroup (not necessarily normal), show that it is possible to find a set of \([G : S]\) of elements that are simultaneously representatives of the right cosets and of the left cosets of \( S \).

43) Show (by constructing a concrete counterexample) that, if sets \( A_1, \ldots, A_{n-1} \subset X \) have an SDR \( a_1, \ldots, a_{n-1} \), and (by adding one more set \( A_n \subset X \)) the enlarged collection \( A_1, \ldots, A_n \subset X \) also affords an SDR, it might not be possible to extend \( a_1, \ldots, a_{n-1} \) (by adding a further element) to an SDR of \( A_1, \ldots, A_n \).