1) If it is 3pm now, what time will it be in 1253 hours?

2) Find by hand integers s and t such that \(1 = 5s + 7t\). Show that s and t are not unique. What is (for a fixed pair \((s, t)\)) the gcd of s and t?

3) A valid CSUID number \(n\) fulfills that \(n \mod 7 = 1\).
   a) Verify this with your own CSUID number.
   b) Find the missing digit \(x\) in the partial CSUID 8765432x.
   c) Give an example of a CSUID number with one unreadable digit that cannot be recovered uniquely.

4) Show that there must be a power of 3 (with exponent \(\geq 1\), i.e. we exclude \(3^0\)) whose last four digits are 0001.
   **Hint:** consider the sequence
   \[
   3^0 \mod 10000, 3^1 \mod 10000, 3^2 \mod 10000, 3^3 \mod 10000, 3^4 \mod 10000, 3^5 \mod 10000, \ldots
   \]
   Show that this sequence must be eventually periodic, i.e. there are \(m < n\) such that \(3^m \mod 10000 = 3^n \mod 10000\). Then, using the invertability of 3 modulo 10000, show that one can go “backwards” from \(3^k\) to \(3^{k-1}\). Conclude that therefore there must be an \(l\), such that \(3^l \mod 10000 = 3^0 = 1\), i.e. the last four digits of \(3^l\) are 0001.

5) a) Solve the equation \((7x + 15) \mod 31 = 14\).
   b) Find (for example by trying out all values of \(x\)) all solutions to the equation \(x^2 \mod 7 = 2\).
   c) Find all solutions to the equation \(x^2 \mod 7 = 3\).
   d) Find all solutions to the equation \(x^2 \mod 8 = 1\).
   e) Based on the results in b-d) (and possibly further experimentation), make a guess about the number of solutions for quadratic equations modulo a number \(m\) (depending on whether \(m\) is a prime).

6) Using the fast powering algorithm, calculate \(2^{477} \mod 1000\)

**Practice Problems:** (These problems from the book are not to be handed in, but might be useful in reviewing and in exam preparation): 1.6, 1.9, 1.10, 1.15, 1.16, 1.17, 1.23