6) The following functions $f_i(n)$ ($1 \leq i \leq 7$) are given:

$$(e^n)^2, 40n^2 - n, 14 \log n^4, 27n^2 + 3n + 1, 14(\log n)^4, 100n \log n, e^{2n},$$

a) Arrange these functions according to $O$ (such that $f_i(n) = O(f_{i+1})$). (Justify your assertions!) (3 Points)

b) Assume these functions are runtimes for 7 algorithms that all solve the same problem. For which values of $n$ (nearest integer) which algorithm has the shortest runtime? (2 Points)

c) Assume you have an algorithm whose runtime is $T(n) = n^2 \log(n)$. For what ranges of $n$ (order of magnitude estimate) this algorithm will run in a second, an hour and a week on a 200MHz Processor, assuming one instruction is executed in one clock cycle. (1 Points)

7) We define 4 procedures $A-D$ as follows:

$A(n)$: for $i \in \{1, \ldots, n\}$ do print "hello"; od;
$B(n)$: for $i \in \{1, \ldots, n\}$ do $A(i)$; od;
$C(n)$: for $i \in \{1, \ldots, n\}$ do $B(i)$; od;
$D(n)$: for $i \in \{1, \ldots, n\}$ do $C(i)$; od;

Give estimates, as $\Theta$ classes, for the run time of each procedure. (4 Points)

8) An (undirected) graph is called a tree if it has only one connected component (i.e. it is connected) and it has no cycles.

Show:

A tree

Not a tree

a) A connected undirected graph $G = (V, E)$ is a tree if and only if $|E| = |V| - 1$.

Hint: You have to show: i) "tree $\Rightarrow |E| = |V| - 1$" and ii) "$|E| = |V| - 1 \Rightarrow$ tree". Use induction on $|V|$. For the induction step, show first that in both cases there must be a vertex of degree 1 (a leaf). (For ii), remember that $2|E| = \sum_{v \in V} \deg(v)$.) Then remove this vertex. (The resulting graph will have one edge and one vertex less and will still be a tree.) (3 Points)

b) A connected undirected graph $G = (V, E)$ is a tree if and only if for two vertices $u, v \in V$ there is exactly one path between $u$ and $v$. (1 Points)

Hand in solutions October 9th before the tutorial.