Sample Exam 1A

Use a #2 pencil. Calculators are allowed, but cell phones/ipods etc. are NOT acceptable. Please turn cell phones off.

Choose the alternative that best completes the statement or answers the question and mark your answer on the scantron form. Scantrons with no answers marked will receive a score of 0.

Use the following preference schedule to answer questions 1 to 8.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>9</th>
<th>6</th>
<th>4</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>First choice</td>
<td>B</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>Second choice</td>
<td>A</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Third choice</td>
<td>D</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Fourth choice</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

1. How many first place votes does candidate C receive?
   
   A) 6
   B) 5
   C) 9
   D) 3
   E) None of the above.

2. What is the minimum number of first place votes needed to have a majority?
   
   A) 23
   B) 11
   C) 12
   D) 9
   E) None of the above.

3. Which candidate has a majority of first place votes?
   
   A) A
   B) B
   C) C
   D) D
   E) None of the above.

4. Which candidate comes in first using the Plurality Method to rank the candidates?
   
   A) A
   B) B
   C) C
   D) D
   E) None of the above.
5. How many points does candidate B earn when using the Method of Pairwise Comparisons?
   A) 3
   B) 2
   C) 1
   D) 0
   E) None of the above.

6. Rank the candidates using the Method of Pairwise Comparisons.
   A) A, B, C, D
   B) A, C, D, B
   C) A, D, B, C
   D) B, D, A, C
   E) C, B, A, D

7. Which candidate is the Condorcet Candidate?
   A) A
   B) B
   C) C
   D) D
   E) None of the above.

8. Which of the following statements is true concerning this preference schedule?
   A) The Majority Criterion is violated by the Method of Pairwise Comparisons.
   B) The Condorcet Criterion is violated by the Method of Pairwise Comparisons.
   C) The Condorcet Criterion is violated by the Plurality Method.
   D) The Majority Criterion is violated by the Plurality Method.
   E) Both the Plurality Method and the Method of Pairwise Comparisons are fair under all 4 criteria.

Use the following preference schedule to answer questions 9 - 12.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>7</th>
<th>5</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>First choice</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>Second choice</td>
<td>B</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>Third choice</td>
<td>D</td>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>Fourth choice</td>
<td>C</td>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>

9. How many points does candidate D earn using the Borda Count method?
   A) 51
   B) 49
   C) 28
   D) 42
   E) None of the above.
10. Rank the candidates using the Borda Count Method.
   A) A, B, C, D
   B) D, A, C, B
   C) B, D, A, C
   D) C, B, D, A
   E) None of the above.

11. Which candidate is eliminated first using Plurality-With-Elimination?
   A) A
   B) B
   C) C
   D) D
   E) None of the above.

12. Rank the candidates using the Plurality-With-Elimination Method.
   A) A, B, C, D
   B) D, A, C, B
   C) B, D, A, C
   D) C, B, D, A
   E) None of the above.

13. Which of the following is NOT a valid quota $q$ for the voting system \([q : 10, 5, 5, 1]\)?
   A) 11
   B) 16
   C) 21
   D) 26
   E) All of the above.

14. In the weighted voting system \([15 : 12, 10, x]\), what is the minimum value for $x$ that guarantees $P_3$ is NOT a dummy voter?
   A) $x = 2$
   B) $x = 3$
   C) $x = 4$
   D) $P_3$ will always be a dummy.
   E) None of the above.
Use the weighted voting system \([13 : 12, 10, 2]\) and the Banzhaf definition of power to answer questions 15 - 19.

15. How many coalitions (not counting the empty coalition) are there?
   A) 6
   B) 7
   C) 8
   D) 9
   E) None of the above.

16. Which players are critical in \(\{12, 2\}\)?
   A) None of the players are critical.
   B) All of the players are critical.
   C) The player with weight 12 is the only critical player.
   D) The player with weight 2 is the only critical player.
   E) None of the above.

17. How many time is the player with weight 10 critical?
   A) 0
   B) 1
   C) 2
   D) 3
   E) None of the above.

18. What is the Banzhaf power distribution?
   A) \(P_1 : \frac{1}{2}; P_2 : \frac{1}{5}; P_3 : \frac{1}{4}\)
   B) \(P_1 : \frac{3}{5}; P_2 : \frac{2}{5}; P_3 : 0\)
   C) \(P_1 : \frac{3}{5}; P_2 : \frac{1}{5}; P_3 : \frac{1}{5}\)
   D) \(P_1 : \frac{3}{7}; P_2 : \frac{2}{7}; P_3 : \frac{2}{7}\)
   E) None of the above.

19. Modify this weighted voting system by changing the quota. For which quota below will Player 1 have veto power?
   A) Player 1 already has veto power.
   B) It is impossible for Player 1 to have veto power.
   C) Change the quota to 14.
   D) Change the quota to 12.
   E) Change the quota to 11.
Use the weighted voting system \([13 : 12, 10, 2]\) and the Shapley-Shubik definition of power to answer questions 20 - 24.

20. Which of the following is a list of all the sequential coalitions?

A) \([12, 10, 2], \langle 12, 10 \rangle, \langle 12, 2 \rangle, \langle 10, 2 \rangle\)
B) \([12, 10, 2], \langle 12, 10 \rangle, \langle 12, 2 \rangle, \langle 10, 2 \rangle, \langle 12 \rangle, \langle 10 \rangle, \langle 2 \rangle\)
C) \([12, 10, 2], \langle 12, 2, 10 \rangle, \langle 2, 10, 12 \rangle, \langle 10, 2, 12 \rangle\)
D) \([12, 10, 2], \langle 12, 2, 10 \rangle, \langle 10, 12, 2 \rangle, \langle 10, 2, 12 \rangle, \langle 12, 10, 2 \rangle, \langle 2, 10, 12 \rangle\)
E) None of the above.

21. What player(s) is/are pivotal in \(\langle 2, 12, 10 \rangle\)?

A) \(P_1\) (12 votes) is the only pivotal player
B) \(P_2\) (10 votes) is the only pivotal player
C) \(P_3\) (2 votes) is the only pivotal player
D) \(P_1\) (12 votes) and \(P_2\) (10 votes) are the only pivotal players
E) None of the above.

22. How many times is the player with weight 2 pivotal? (What is the pivotal count?)

A) 0
B) 1
C) 2
D) 3
E) None of the above.

23. What is the Shapley-Shubik power distribution for the voting system?

A) \(P_1 : \frac{1}{2}; P_2 : \frac{1}{5}; P_3 : \frac{1}{4}\)
B) \(P_1 : \frac{2}{3}; P_2 : \frac{1}{6}; P_3 : \frac{1}{6}\)
C) \(P_1 : \frac{1}{3}; P_2 : \frac{1}{5}; P_3 : \frac{1}{3}\)
D) \(P_1 : \frac{2}{6}; P_2 : \frac{1}{6}; P_3 : 0\)
E) None of the above.

24. If the number of players increases to 10, how many sequential coalitions would there be?

A) \(2^{10}\)
B) \(2^{10-1}\)
C) \((10 - 1)!\)
D) \(10!\)
E) None of the above.