Math 113 HW #1 Solutions

§ 1.1

6: Determine whether the curve is the graph of a function of $x$. If it is, state the domain and range of the function.

**Answer:** The pictured curve is the graph of a function. The domain and range of the function are:

- **Domain:** $-2 \leq x \leq 2$
- **Range:** $-1 \leq y \leq 2$

23: Given $f(x) = 4 + 3x - x^2$, evaluate the difference quotient

$$\frac{f(3 + h) - f(3)}{h}.$$ 

**Answer:** Plugging in $x = 3 + h$ to $f(x)$ yields

$$f(3 + h) = 4 + 3(3 + h) - (3 + h)^2 = 4 + 9 + 3h - (9 + 6h + h^2) = 4 - 3h - h^2.$$ 

Likewise,

$$f(3) = 4 + 3(3) - 3^2 = 4.$$ 

Therefore, the difference quotient

$$\frac{f(3 + h) - f(3)}{h} = \frac{(4 - 3h - h^2) - 4}{h} = \frac{-3h - h^2}{h} = -3 - h.$$ 

44: Find the domain and sketch the graph of the function

$$f(x) = \begin{cases} 
  x + 9 & \text{if } x < -3 \\
  -2x & \text{if } |x| \leq 3 \\
  -6 & \text{if } x > 3 
\end{cases}.$$ 

**Answer:** Since the three pieces in the definition of $f$ account for all real numbers, the domain of $f$ consists of all real numbers. The graph of $f$ is shown in Figure 1.
56: A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area $A$ of the window as a function of the width $x$ of the window.

**Answer:** Let $h$ denote the height of the rectangle. Then we know that the perimeter of the window is equal to

\[ x + 2h + \text{outer perimeter of semi-circle}. \]

Since the semi-circle in our Norman window has radius $x/2$, its contribution to the perimeter of the window is half the circumference of a circle of radius $x/2$:

\[ \frac{1}{2} \left( 2\pi \frac{x}{2} \right) = \frac{\pi x}{2}. \]

Therefore, the perimeter of the window is

\[ x + 2h + \frac{\pi x}{2} = \left( 1 + \frac{\pi}{2} \right) x + 2h. \]

Since we know the perimeter of the window is equal to 30 ft, the above expression is equal to 30 and we can solve for $h$:

\[ 2h = 30 - \left( 1 + \frac{\pi}{2} \right) x, \]

so

\[ h = 15 - \left( \frac{1}{2} + \frac{\pi}{4} \right) x. \]
Therefore, the area $A$ of the window is equal to

$$A(x) = \text{area of rectangle} + \text{area of semi-circle}$$

$$= xh + \frac{1}{2} \pi \left( \frac{x}{2} \right)^2$$

$$= x \left[ 15 - \left( \frac{1}{2} + \frac{\pi}{4} \right) x \right] + \frac{\pi x^2}{8}$$

$$= 15x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$

$$= 15x - \left( \frac{1}{2} + \frac{\pi}{8} \right) x^2.$$

\[\text{§ 1.2}\]

2: Classify each of the following functions:

(a) $y = \frac{x - 6}{x + 6}$ is a rational function.

(b) $y = x + \frac{x^2}{\sqrt{x-1}}$ is an algebraic function.

(c) $y = 10^x$ is an exponential function.

(d) $y = x^{10}$ is a polynomial of degree 10.

(e) $y = 2t^6 + t^4 - \pi$ is a polynomial of degree 6.

(f) $y = \cos \theta + \sin \theta$ is a trigonometric function.

4: Match each equation with its graph

(a) $y = 3x$ corresponds to the graph $G$.

(b) $y = 3^x$ corresponds to the graph $f$.

(c) $y = x^3$ corresponds to the graph $F$.

(d) $y = \sqrt{x}$ corresponds to the graph $g$.

6: What do all the members of the family of linear functions $f(x) = 1 + m(x + 3)$ have in common? Sketch several members of the family.

\textbf{Answer:} Each of the functions in this family is a line passing through the point $(-3, 1)$. By varying the different values of $m$ we can get all such lines except the vertical line (which would correspond to $m = \infty$, if that was a valid choice for $m$). Several members of this family of lines are shown in Figure 2.
16: The manager of a furniture factory finds that it costs $2200 to manufacture 100 chairs in one day and $4800 to produce 300 chairs in one day.

(a) Express the cost as a function of the number of chairs produced, assuming that it is linear. Then sketch the graph.

**Answer:** First, it makes sense to think of the number of chairs as the input and the cost to produce them as the output. Therefore, let $C(x)$ denote the cost of producing $x$ chairs. Assuming $C(x)$ is linear, we want to find the equation of the line passing through the points $(100, 2200)$ and $(300, 4800)$. Such a line has slope

$$m = \frac{4800 - 2200}{300 - 100} = \frac{2600}{200} = 13.$$ 

Therefore, using the point-slope formula, the equation of the line is

$$y - 2200 = 13(x - 100),$$

so

$$y = 13(x - 100) + 2200 = 13x - 1300 + 2200 = 13x + 900.$$ 

Thus, we see that

$$C(x) = 13x + 900.$$
(b) What is the slope of the graph and what does it represent?
   Answer: The slope of the graph $y = C(x)$ is equal to 13; this represents the cost of producing an additional chair. In economic terms, the marginal cost of production is $13/\text{chair.}$

(c) What is the $y$-intercept of the graph and what does it represent?
   Answer: The $y$-intercept of $y = C(x)$ is equal to $900; this represents the fixed costs of production.