DIFFERENTIAL GEOMETRY HW 6

CLAY SHONKWILER

(1) Is it possible to make the double torus (i.e. orientable surface of genus 2) into a Lie group?
(2) Give an example of a Riemannian metric on a Lie group which is not bi-invariant. Is it possible to put a bi-invariant metric on this Lie group?
(3) Describe the geodesics in $SO(n)$.
(4) Is the product metric on $S^3 \times S^1$ bi-invariant?
(5) What are the Gauss-Kronecker and mean curvatures of an immersed hypersurface $N$ of a manifold $M$? Show that if the Gauss-Kronecker curvature vanishes somewhere and $M$ is hyperbolic space then $N$ cannot be flat.
(6) What does it mean for a submanifold to be minimal? Give an example of a minimal submanifold of $\mathbb{R}^3$.
(7) Can you put a metric on $S^2 \subset \mathbb{R}^3$ so that the identity inclusion is geodesic at a point $p \in S^2$? Is there a metric on $S^2$ so that it is totally geodesic in $\mathbb{R}^3$?
(8) Let $p \in \mathbb{CP}^2$ and consider the 2-sphere $S^2$ which is the cut locus of $p$. Is $S^2$ a minimal submanifold of $\mathbb{CP}^2$? Is it totally geodesic?
(9) Is $M$ minimal in $TM$? Totally geodesic?
(10) Suppose $N$ is a submanifold of $M$ such that $TN$ is a totally geodesic submanifold of $TM$. Is $N$ totally geodesic in $M$? What about the converse?
(11) Find a totally geodesic submanifold of $SO(5)$ of dimension $\geq 2$.
(12) What is the conjugate locus of a point $p \in S^2 \times S^2$?
(13) Describe the focal locus of the catenoid.
(14) If $M$ and $N$ are Riemannian manifolds, is "$M$ is locally isometric to $N$" a symmetric relation?
(15) Is it possible for a submanifold $N \subset M$ to be its own focal locus?