M161, Midterm 3, Fall 2010

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Name:________________________

Section:_____________________

Instructor:__________________

Time: 75 minutes. You may not use calculators or other electronic devices on this exam

\[
\frac{d}{dx} \sin(x) = \cos(x), \quad \frac{d}{dx} \cos(x) = -\sin(x), \quad \frac{d}{dx} \tan(x) = \sec^2(x),
\]
\[
\frac{d}{dx} \sin(x) = \cos(x), \quad \frac{d}{dx} \cos(x) = -\sin(x), \quad \frac{d}{dx} \tan(x) = \sec^2(x),
\]
\[
\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}},
\]
\[
\frac{d}{dx} \arccsc(x) = -\frac{1}{x\sqrt{x^2-1}}, \quad \frac{d}{dx} \arcsec(x) = \frac{1}{x\sqrt{x^2-1}},
\]
\[
\sin(2x) = 2\sin(x)\cos(x) \quad \int \ln x\,dx = x\ln x - x + C \quad \int \sec(x)\,dx = \ln |\sec(x) + \tan(x)| + C
\]
\[
\tan^2(x) + 1 = \sec^2(x) \quad \cos^2(x) = \frac{1 + \cos(2x)}{2} \quad \sin^2(x) = \frac{1 - \cos(2x)}{2}
\]

Taylor series of \(f(x)\) about \(x = a\):

\[
f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.
\]
1. Short answer questions. Put your answer in the box. No work outside the box will be graded.

(a) Express the number $0.\overline{34} = 0.343434\ldots$ as a fraction in lowest terms.

(b) What are the center and radius of the interval of convergence of the power series
\[
\sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n}
\]?

Center: \[\] Radius of convergence: \[\]

(c) The 4th order Taylor polynomial of $f(x)$ centered at $a = 0$ is
\[
1 + x - 3x^2 + 4x^3 - 5x^4.
\]

Is $f(x)$ increasing or decreasing near $x = 0$? \[\]

Is $f(x)$ concave up or concave down near $x = 0$? \[\]
2. For each of the following series, state whether it converges or diverges and give a short justification.

(a) \( \sum_{n=0}^{\infty} \frac{n^4}{4^n} \):

(b) \( \sum_{n=0}^{\infty} \cos(\pi n) \):

(c) \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n^2} \):
3. Match the following functions to their corresponding power series:

______ $e^{2x} - 1$

______ $\frac{8x}{4-2x}$

______ $2xe^{2x}$

______ $2x \frac{1 - 2^{10}x^{10}}{1 - 2x}$

______ $\arctan(2x) + \frac{1}{2} \ln(1 + 4x^2)$

______ $\sin(2x) + \cos(2x) - 1$

A. $\sum_{n=1}^{\infty} \frac{x^n}{2n-2}$

B. $\sum_{n=1}^{10} 2^n x^n$

C. $\sum_{n=1}^{\infty} \frac{2^n}{(n-1)!} x^n$

D. $\sum_{n=1}^{\infty} \frac{2^n}{n!} x^n$

E. $\sum_{n=1}^{\infty} (-1)^{\frac{n^2+n+2}{2}} \frac{2^n}{n!} x^n$

F. $\sum_{n=1}^{\infty} (-1)^{\frac{n^2+n+2}{2}} \frac{2^n}{n} x^n$
4. The Taylor series for \( \frac{1}{1-x} \) is \( \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots \).

(a) The partial sum \( s_n(x) = 1 + x + \cdots + x^n \) equals \( \frac{1 + t_n(x)}{1-x} \) for some function \( t_n(x) \). What is \( t_n(x) \)?

(b) For what values of \( x \) does \( \lim_{n \to \infty} s_n(x) \) exist?

(c) Complete this sentence: If the sequence of partial sums of a series has a finite limit, then the series \underline{______}.

(d) Use the Taylor series for \( \frac{1}{1-x} \) to find the Taylor series for \( \frac{1}{(1-x)^2} \).
5. Let \( f(x) = \cos(2x) \) and let \( a = \pi/8 \).

(a) Compute the degree 3 Taylor approximation \( T_3(x) \) for \( f(x) \) at 
\( x = a \). (This is the one that ends with a constant times \( x^3 \)).

(b) What is the coefficient of \( x^{2010} \) in the Taylor series for \( f(x) \) centered 
at \( x = a \)?

(c) Complete the sentence: The polynomial \( T_3(x) \) is the best __________ for \( f(x) \) near \( x = a \).