M501 Combinatorics I

exercise sheet # 4

Exercise # 1 (2 points)
In how many ways can we distribute 23 apples to Fred, Jane and George?

Exercise # 2 (2 points)
Given 3 white balls and one black ball, what is the number of ways to distribute them into two square boxes and one round box (assuming one cannot distinguish between the two square boxes)?

Exercise # 3 (3 points)
A (simple) graph is called self-complementary if it is isomorphic to its complement (i.e. the graph with edges only where the original graph had no edge). Construct the self complementary graphs with 4, 5 and 6 vertices. Determine their automorphism groups.

Exercise # 4 (3 points)
Let $G$ be a finite group acting transitively on the finite set $X$ (i.e. there is only one orbit). Show that for arbitrary $x \in X$ we have

$$|G_x \backslash X| = \frac{1}{|G|} \sum_{g \in G} |X_g|^2.$$