You may NOT use calculators or any references. Show work to receive full credit.

Recall:
\[
\int \cos^2(at)\,dt = \frac{1}{2} \left( t + \frac{1}{2a} \sin(2at) \right) + C
\]

and
\[
\int \sin^2(at)\,dt = \frac{1}{2} \left( t - \frac{1}{2a} \sin(2at) \right) + C
\]

GOOD LUCK!!!
1. Given the equations of the three planes

\[ P_1: \ x + y + z = 4 \]
\[ P_2: \ 2x + 2y + 2z = 6 \]
\[ P_3: \ x - y + z = 2 \]

and the vector equations of the two lines

\[ L_1: \ \mathbf{r}_1(t) = \langle t, 1, 3 - t \rangle \]
\[ L_2: \ \mathbf{r}_2(t) = \langle t, t, t \rangle \]

circle TRUE or FALSE for the following four questions (do not show work):

(6 pts) TRUE or FALSE: P1 and P2 never intersect.
(6 pts) TRUE or FALSE: Line L1 is the intersection of two of the planes.
(6 pts) TRUE or FALSE: Lines L1 and L2 intersect.
(6 pts) TRUE or FALSE: Plane P3 and line L2 never intersect.

2. Considering \( \mathbf{a}(t) = 6t^2 \mathbf{i} - \frac{1}{t^2} \mathbf{j} + e^t \mathbf{k} \), complete the following.

(a) (12 pts) Given that \( \mathbf{v}(1) = \mathbf{i} + 3 \mathbf{j} + \mathbf{k} \), find \( \mathbf{v}(t) \).
(b) (12 pts) Given that \( \mathbf{r}(1) = \mathbf{i} + 2 \mathbf{j} + 3 \mathbf{k} \), find \( \mathbf{r}(t) \).

3. Consider the surface \( x^2 \cos(z) + ye^{3y} = e^3 \).

(a) (4 pts) Check that the point \( P(1, 1, \pi/2) \) is a point on the surface.
(b) (12 pts) Find an equation of the tangent plane at \( P \).
(c) (9 pts) Find a vector equation of the line normal to the surface at \( P \).

4. Consider the function \( f(x,y) = x^3 + 3xy^2 - 6x^2 - 6y^2 + 2 \)

(a) (10 pts) One of the critical points is \( (0,0) \). Classify it.
(b) (12 pts) Find the remaining critical points. Express them in the form \( (x,y) \).

Do NOT classify them. Circle the answers.

5. (24 pts) Find the area of the planar region bounded on the inside by the circle \( r = 1 \) and on the outside by the limaçon \( r = 2 + \cos \theta \). Include a graph of the region in the \( xy \) plane.

6. Consider the solid bounded by the surfaces \( z = 2 - x, \ x = y^2, \) and \( z = 0 \).

(a) (10 pts) Graph the solid, clearly labeling all corners.
(b) (6 pts) Set up the integral representing volume with \( dV = dzdxdy \). Do NOT integrate.
(c) (6 pts) Repeat (b) with \( dV = dydzdx \).
(d) (6 pts) Repeat (b) with $dV = dx dy dz$.

7. (18 pts) Using Green’s Theorem and the appropriate choice of variables (i.e., NOT CARTESIAN), evaluate $\int_C y^2 dx + 3xydy$ where $C$ is the boundary of the region in the upper half plane bounded by $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and $y = 0$. Be sure to clearly identify the integral you evaluate.

8. Consider the surface $S$ which is the portion of the cone $y^2 = x^2 + z^2$ between $y = 1$ and $y = 3$ and the function $G(x, y, z) = z$.

a. (4 pts) Sketch the surface.

b. (7 pts) Parametrize the surface using parameters $r$ and $\theta$, including lower and upper bounds on both parameters.

c. (8 pts) Set up the integral

$$\int_S G(x, y, z) d\sigma,$$

clearly indicating the bounds of integration and all parts of the integrand, but DO NOT EVALUATE.

9. Using Stokes’ theorem, set up a surface integral that computes the circulation of vector field $\mathbf{F} = \langle z, \frac{1}{2}x^2, y \rangle$, where $C$ is the curve of intersection of the plane $y + z = 1$ and the ellipsoid $2x^2 + y^2 + z^2 = 1$, as shown in the figure below. Use the direction of travel around $C$ that is counterclockwise to the outward normal to the sphere. Do this in the following steps:

(a) (3 pts) Parameterize the plane $y + z = 1$ using $x$ and $z$ as parameters.

(b) (4 pts) Calling the parameterization $\mathbf{r}(x, z)$, compute $\mathbf{r}_x \times \mathbf{r}_z$.

(c) (4 pts) Sketch the projection of the part of the plane $y + z = 1$ bounded by the given curve $C$ onto the $xz$-plane.

(d) (5 pts) Set up the surface integral, but do NOT evaluate the integral.