Pries: 470 Euclidean and non-Euclidean Geometry

Homework 8: Euclidean Transformations
Due Friday March 10

More on Euclidean Transformations:

1. Find the images of $P = (0, 0)$, $Q = (0, 2)$, and $R = (2, 1)$ under the clockwise rotation by 45 degrees that fixes $(3, 3)$:
   a. using the rotation matrix.
   b. using complex numbers.

2. A set $S$ of points has rotational symmetry if it is invariant under some rotation. How many rotational symmetries do these sets have?
   a. an isosceles triangle.
   b. a regular pentagon.
   c. a regular hexagon.
   d. a circle.

3. Let $m = \tan(\theta)$. Let $\gamma_{\ell}$ be the reflection over the line $\ell : y = mx$.
   a. Write $\gamma_{\ell}$ as the composition of three transformations.
   b. Now $\gamma_{\ell} = T_C \circ \rho_B$ for some $C$ and some unit vector $B$. Use the composition rules to find $C$ and $B$.
   c. Find a formula for $\gamma_{\ell}(z)$ using complex numbers.

4. There is some isometry $T = T_C \circ \rho_B \circ \gamma$ that takes $\triangle ABC$ to $\triangle DEF$ where $A = (6, 2)$, $B = (7, 2)$, $C = (7, 4)$, $D = (-4, 3)$, $E = (-7/2, 3 + \sqrt{3}/2)$, and $F = (-4 - \sqrt{3}, 4)$. What are $C$ and $B$?

5. Let $\gamma_{\ell}$ be the reflection over the line $\ell$.
   a. Find two lines $\ell$ and $m$ so that $\gamma_{\ell} \circ \gamma_m = \rho_{\pi/4}$.
   b. Find two lines $\ell$ and $m$ so that $\gamma_{\ell} \circ \gamma_m = T_{(4, 4)}$.

6. Find the images of $(0, 0)$, $(0, 2)$ and $(2, 1)$ under the glide reflection $G_{AB}$ where $A = (-2, 0)$ and $B = (0, 1)$. 