Final Exam

11:50AM-1:50PM, May 14, 2019

Name: ________________________ Instructor: __________ Time your class meets: _____

HONOR PLEDGE I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

Signature: ____________________ Date: _______________

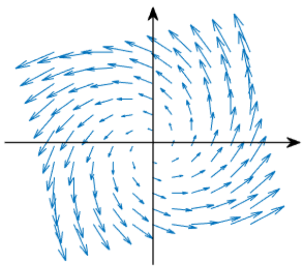
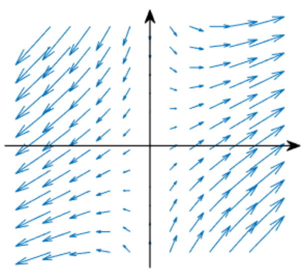
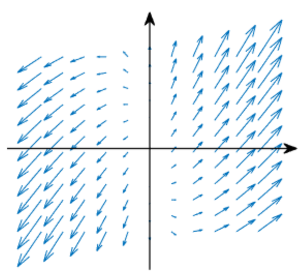
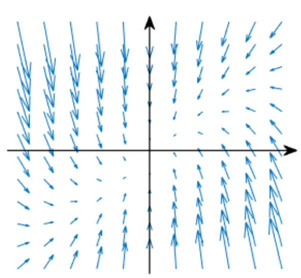
- You have one hour and fifty minutes to complete this exam.
- No notes, books, or other references are allowed during this exam.
- Calculators are not allowed during the exam.
- A two-sided cheat sheet of $8\frac{1}{2}'' \times 11''$ or smaller is allowed during the exam.
- There are questions on the front of the page. Answers must be written in the exam, and you may use the back of each page if you need more space.
- You must show all work to receive credit. Answers for which no work is shown will receive no credit unless specifically stated otherwise.

Question	Score	Maximum
1		12
2		15
3		10
4		13
5		15
6		20
7		15
Total		100

1. (**12 points**) We have five systems of linear ODEs of the form $\mathbf{x}' = A_i\mathbf{x}$ with different A_i:

$$A_1 = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix};\ A_2 = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix};\ A_3 = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix};\ A_4 = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix};\ A_5 = \begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix}.$$

For each of the following four vector fields, **state the type** (Nodal sink, Nodal source, Center, Saddle, Spiral sink, or Spiral source) **and stability** of the equilibirum point at the origin, and choose the right linear system for the vector field.

Solution vector field	Type	Stability	$\mathbf{x}' = A_i\mathbf{x}$ with which A_i?
			
			
			
			

2. (**15 points**) Consider the first-order differential equation

$$(xy + 1)dx + (x^2 + xy)dy = 0.$$

(a) Show that the differential equation is not exact.

(b) This differential equation admits an integrating factor that depends only on x. Find such an integrating factor.

(c) Using the integrating factor to solve for a general solution to the differential equation.

3. (**10 points**) Solve the following inhomogeneous second order equation using characteristic polynomial and undetermined coefficients:

$$y'' - 2y' - 3y = 5\cos(2t).$$

4. (**13 points**) Use Laplace transforms to solve the initial value problem

$$x'' - 6x' + 8x = 2, \quad x(0) = x'(0) = 0.$$

5. (**15 points**) Consider the matrix

$$A = \begin{bmatrix} 1 & -4 & 8 \\ 0 & 1 & 8 \\ 0 & 0 & 5 \end{bmatrix}$$

(a) Find the eigenvalues of A.

(b) Find the eigenvectors of A.

(c) What are the algebraic and geometric multiplicities of the eigenvalues?

(d) One of the eigenvalues has an algebraic multiplicity strictly greater than its geometric multiplicity. Find a generalized eigenvector for that eigenvalue.

(e) Give the fundamental set of solutions to the system $\mathbf{x}' = A\mathbf{x}$.

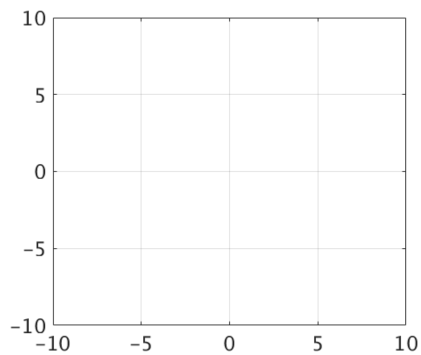
(f) Give the particular solution to $\mathbf{x}' = A\mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$.

6. (**20 points**) Consider the second order equation

$$y'' + 2y'y = 9y - y^2.$$

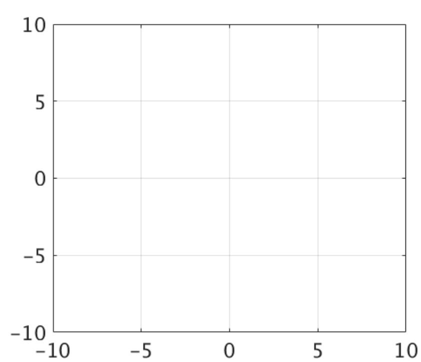
(a) Write the second order equation as a system of first order equations.

(b) Find all nullclines and equilibria for the system from (a). Sketch and label each.



(c) Linearize the system about the equilibrium point at the origin using Jacobian.

(d) Use the linearization to classify the equilibrium point at the origin and sketch the phase portrait.



7. (**15 points**) Consider the linear system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 4 \\ 2x_1 - 2x_2 - 2x_3 + 5x_4 &= a^2 \\ -x_1 - 8x_2 - 11x_3 - 7x_4 &= a \end{aligned}$$

(a) Write the system as an augmented matrix.

(b) Convert the augmented matrix into row echelon form.

(c) Determine all the values of a so that the linear system is consistent.

(d) Find the solution of the linear system for the largest a in Part (c) and write the solution in parametric form.