EXAM II

5:00-6:50PM, Apr 18, 2019

Name: ________________________ Instructor: __________ Time your class meets: _____

HONOR PLEDGE I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

Signature: ____________________ Date: _______________

- You have one hour and fifty minutes to complete this exam.
- No notes, books, or other references are allowed during this exam.
- Calculators are not allowed during the exam.
- A two-sided cheat sheet of $8\frac{1}{2}'' \times 11''$ or smaller is allowed during the exam.
- There are questions on the front of the page. Answers must be written in the exam, and you may use the back of each page if you need more space.
- You must show all work to receive credit. Answers for which no work is shown will receive no credit unless specifically stated otherwise.

Question	Score	Maximum
1		15
2		15
3		15
4		15
5		15
6		10
7		15
Total		100

1. (**15 points**) Short answer problems.

(a) Let a non-zero square matrix $A \in \mathbb{R}^{n\times n}, n > 1$ be nonsingular. Determine whether the following statements for A is Ture or False by respectively marking tick or cross in the box before each statement.

- ☐ $Ax = 0$ has nontrivial solution.
- ☐ $Ax = b$ is inconsistent for some b.
- ☐ A is invertible.
- ☐ Colume vectors of A are linearly dependent.
- ☐ A has an eigenvalue being 0.
- ☐ Null space of A is nontrivial.
- ☐ Solution of $Ax = b$ does not have a free variable for any b.
- ☐ Row echelon form of A has at least one zero row.
- ☐ A has a zero column.

(b) Determine whether equation

$$(x-1)y'' - 2xy' + (x+1)y = 0, \quad x > 1$$

is linear or nonlinear. Then write this equation into a system of first order equations.

(c) Use the derivative property of the Laplace transform to find the Laplace transform of $y(t) = t^3$.

2. (**15 points**) Consider the linear system

$$\begin{aligned} x_1 + 4x_2 + 6x_3 &= 7 \\ x_1 + 2x_3 &= 3 \\ -x_1 - 2x_2 - 4x_3 &= 5 \end{aligned}$$

(a) Write the system as an augmented matrix.

(b) Reduce the augmented matrix into row echelon form.

(c) Find the solution of the linear system and write it in parametric form.

3. (**15 points**) Consider the matrix

$$A = \begin{bmatrix} 2-x & 0 & 0 \\ 1 & 9-x & 0 \\ 4 & 1 & 8-x \end{bmatrix}.$$

(a) Find the determinant of A.

(b) For what values of x is the matrix A not invertible?

(c) Let $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$. Compute the matrix-vector product Av.

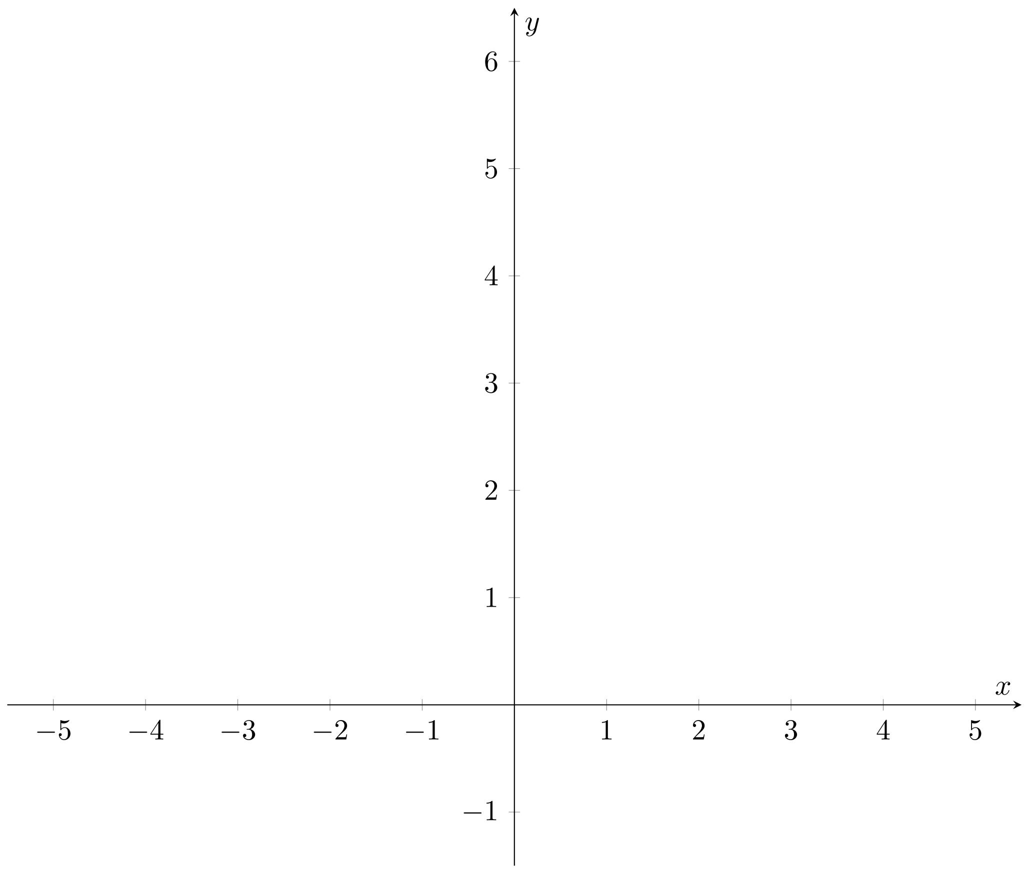
(d) For what values of x is $\vec{v}$ in the nullspace of A?

4. (**15 points**) Consider the following system of equations:

$$\begin{cases} x' = 9x - 8xy \\ y' = y^2 - \frac{1}{2}x^2 y \end{cases}$$

(a) Sketch the nullclines for this system in the phase plane. Use different colors or line styles for the x-nullcline and y-nullcline (For example, dotted lines and solid lines).

(b) Find any equilibrium solutions and identify them on your sketch in the phase plane.



5. (**15 points**) Consider a matix

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{1}$$

(a) Reduce A into a row-echelon form.

(b) Write the solution set of $Ax = 0$ in a parametric form.

(c) Present a set of basis of the null space of A and state the dimension of this space.

(d) Write the vector $x = (-3, 1, -2, 0, 2)^T$ as a linear combination of basis of the null space of A.

6. (**15 points**) Use the Laplace transform to solve the following differential equation:

$$y'' + 4y' + 8y = e^{2t}, \quad y(0) = 2, \quad y'(0) = 0.$$

7. (**15 points**)

(a) Compute the determinant of $A = \begin{bmatrix} 2 & -8 \\ -3 & 12 \end{bmatrix}$.

Now, use the value of $\det(A)$ to **check all boxes that apply** to the matrix.

- ☐ The columns of A form a linearly independent set.
- ☐ A is invertible.
- ☐ $\text{Null}(A) = \left\{\vec{0}\right\}$.
- ☐ None of the above

(b) Compute the determinant of $B = \begin{bmatrix} -2 & 3 & 5 \\ 2 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix}$. Be sure to show all of your work.

Now, use the value of $\det(B)$ to **check all boxes that apply** to the matrix.

- ☐ The columns of B form a linearly independent set.
- ☐ B is invertible.
- ☐ $\text{Null}(B) = \left\{\vec{0}\right\}$.
- ☐ None of the above

(c) The determinant of the matrix shown below is 0. State briefly why this is the case **without doing any calculations.**

$$C = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 2 & 4 & -2 & -4 \\ 3 & 6 & -3 & -6 \\ 4 & 8 & -4 & -8 \end{bmatrix}$$