EXAM I

5:00-6:50PM, Mar 7, 2019

Name: ______________________ Instructor: __________ Time your class meets: _____

HONOR PLEDGE I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

Signature: ____________________ Date: ______________

- You have one hour and fifty minutes to complete this exam.
- No notes, books, or other references are allowed during this exam.
- Calculators are not allowed during the exam.
- A two-sided cheat sheet of $8\frac{1}{2}'' \times 11''$ or smaller is allowed during the exam.
- There are questions on the front of the page. Answers must be written in the exam, and you may use the back of each page if you need more space.
- You must show all work to receive credit. Answers for which no work is shown will receive no credit unless specifically stated otherwise.

Question	Score	Maximum
1		15
2		15
3		15
4		20
5		20
6		15
Total		100

1. (**15 points**) Short answer problems.

(a) Find the equilibrium points and determine their stability for the differential equation $y' = y^3 - 9y$.

(b) Put the following ordinary differential equations into all applicable categories below.

(a)	$y' = y^2 - t$	(b)	$tx' = x$
(c)	$y' - 4y = e^{-3t}$	(d)	$(2x + y)dx + (x - 6y)dy = 0$
(e)	$y' = \cos(ty)$	(f)	$yy'' + t^2 y = \sin(t)$
(g)	$y' = x^2$	(h)	$x' = \dfrac{2tx}{1 + x}$

Separable equations: ____________________________

First order linear equations: ______________________

Exact equations: ________________________________

Nonlinear equations: ____________________________

None of the above: _____________________________

(c) The differential equation $y'' - 3y' - 4y = 0$ has a general solution of

$$y(t) = C_1 e^{-t} + C_2 e^{4t}.$$

What is the particular solution to this differential equation such that $y(0) = 5$ and $y'(0) = 10$?

2. (**15 points**) Consider first order differential equation

$$t^2y' = 2ty + t^4\cos(4t).$$

(a) Find its general solution.

(b) Find its particular solution such that $y(\pi) = 1$.

3. (**15 points**) Consider a first-order ODE:

$$(4xy + 2y\cos(x))dx + (3x^2 + 3\sin(x))dy = 0.$$

(a) Show that this differential equation is not exact;

(b) This differential equation has an integrating factor that depends on y only. Find such an integrating factor;

(c) Find the general solution of this differential equation.

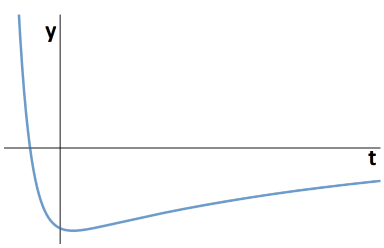
4. (**20 points**) Consider the differential equation $y'' - 2y' + 3y = 0$.

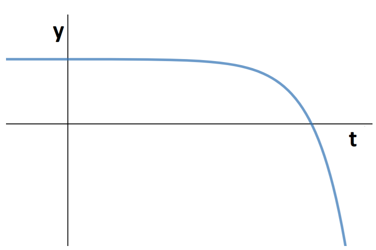
(a) What is the characteristic polynomial of this differential equation?

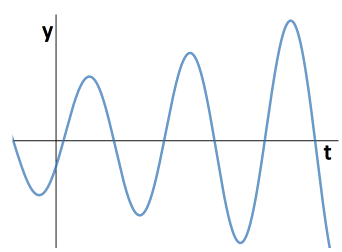
(b) Determine the root(s) of this characteristic polynomial.

(c) What is the **real-valued** general solution to this differential equation?

(d) Circle the graph that most accurately resembles the typical behavior of the real-valued general solution.







5. (**20 points**) Consider a lake that is stocked with walleye pike and that the population of pike is governed by the logistic equation

$$P' = f(P) = 0.1P\left(1 - \frac{P}{10}\right)$$

where time is measured in days and P **in thousands of fish**.

(a) Use the graph of $f(P)$ to develop a phase line for the autonomous equation $P' = f(P)$. Classify each equilibrium point as either unstable or asymptotically stable.

(b) Sketch the equilibrium solutions in the tP-plane. These equilibrium solutions divide the tP-plane into regions. Sketch at least one solution trajectory in each of these regions.

(c) What will happen to $P(t)$ if the initial population is 7500 fish?

(d) Now consider a new scenario in which 160 fish are removed each day. Modify the logistic model to account for the fishing to get a new differential equation $P' = g(P)$. What shall the function $g(P)$ be?

(e) Sketch the equilibrium solutions in the tP-plane for this new scenario. These equilibrium solutions divide the tP-plane into regions. Sketch at least one solution trajectory in each of these regions. (Hint: $P^2 - 10P + 16 = (P - 2)(P - 8)$.)

(f) If the initial fish population is 1000, what happens to the fish as time passes?

6. (**15 points**) Consider the first-order differential equation $y' = \dfrac{y^{2/3}}{t^2 - 4}$.

(a) Find all initial conditions (t_0, y_0) for which the equation is NOT guaranteed to have a unique solution.

(b) Show that $y(t) = 0$ is a solution to the equation with the initial condition $y(0) = 0$.

(c) Assume that $y(t)$ is not constantly zero in each of the following intervals of t. Determine whether $y(t)$ is increasing or decreasing in the interval:

$y(t)$ is __________ in $(-\infty, -2)$;
$y(t)$ is __________ in $(-2, 2)$;
$y(t)$ is __________ in $(2, \infty)$.

(d) Now suppose that $y(t)$ is a solution to the equation with the initial condition $y(0) = 1$. Based on the uniqueness theorem, which of the following are true statements? (Select all that apply.)

- $y(t) = 0$ is possible at $t = -2$.
- $y(t) = 0$ is possible at $t = -1$.
- $y(t) = 0$ is possible at $t = 0$.
- $y(t) = 0$ is possible at $t = 1$.
- $y(t) = 0$ is possible at $t = 2$.
- $y(t) = 0$ is not possible for any t.