# MATH495.3 (CRN 13695)

## Laboratory # 6 : Stage-Classified Matrix Models

Reference Caswell, Hal. *Matrix Population Models: construction, analysis, and interpretation. 2nd ed.* Sinauer Associates, Inc. 2001. p56-62.

# Introduction

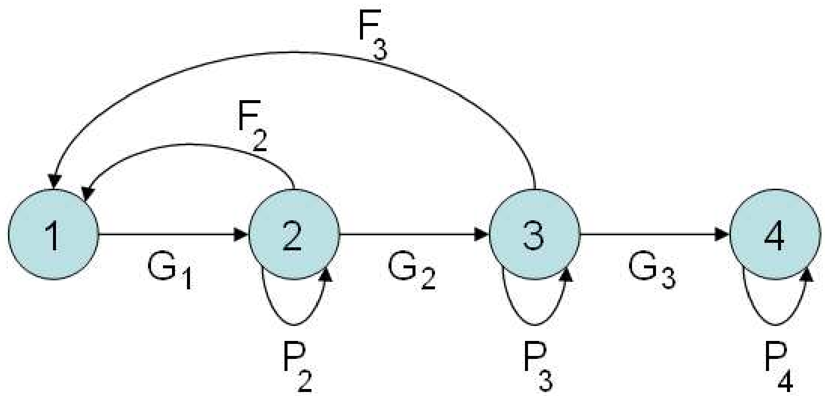
Stage-classified population models are constructed by classifying a given plant or animal's life in to discrete stages, e.g. yearling, juvenile, sub-adult, adult, etc. The $s$ be the number of stages and let the state vector $\boldsymbol{x}(t) \in \mathbb{R}^s$ contain the density of individuals in each stage at each time $t$, i.e., the number of individuals per unit area in each stage at each time $t$. Perhaps the most common form of population models are linear, discrete time models. Their popularity has spawned special terminology. A "Leslie matrix" describes the transitions between stages, and has a particularly simple form involving only the probability of surviving and staying in a given stage ($P_i$), the probability of surviving and growing to the next stage ($G_i$), and the fertility of a given stage ($F_i$). Given initial population density, $\boldsymbol{x}(0)$, for $t = 0, 1, 2, \ldots$, let

$$\boldsymbol{x}(t+1) = \boldsymbol{A}\ \boldsymbol{x}(t) = \quad \text{where} \quad \boldsymbol{A} = \begin{pmatrix} P_1 & F_2 & F_3 & F_4 \\ G_1 & P_2 & 0 & 0 \\ 0 & G_2 & P_3 & 0 \\ 0 & 0 & G_3 & P_4 \end{pmatrix} \tag{1}$$

or, componentwise for $i = 1, \ldots, s$,

$$x_i(t+1) = \sum_{j=1}^{s} a_{ij} x_j(t).$$

Consider the *life cycle graph* for the killer whale, *Orcinus orca.* Nodes represent the following stages: 1; yearlings; 2, juveniles; 3, mature females; and 4, post-reproductive females. The interval between estimates of the population (time step) is one year. In the figure below $P_i$ is the probability of surviving and staying in stage $i$, $G_i$ is the probability of surviving and growing from stage $i$ to stage $i + 1$, and $F_i$ is the fertility of stage $i$.




Note:

1. Because stage 1 is an age class of the same length as the projection interval, individuals cannot remain in stage 1 from one time to the next. Thus, $P_1 = 0$ and no self-loop has been drawn on $x_1$.

2. There is a post-reproductive stage, which does not contribute individuals to any of the other stages.

3. We have positive fertility for juvenile females. This arises because we have assumed that some juveniles may mature during the interval from $t$ to $t+1$ and produce offspring at $t+1$.

The construction of a life cycle graph should, in general, proceed as follows:

1. Choose the stages $1, \ldots, s$ to describe the life cycle model

2. Choose a projection time step

3. Create a node for each stage, numbered from 1 to $s$

4. Place a directed line (arc) from stage $i$ to stage $j$ if an individual in stage $i$ at time $t$ can contribute individuals (by development or reproduction) to stage $j$ at time $t+1$ (Note: It is possible that $j = i$)

5. Determine the coefficient $a_{ji}$ on the arc from $i$ to $j$ describing the contribution of stage $i$ at time $t$ to stage $j$ at time $t+1$.

## Assignment

Construct the stage-classified model for six stages of the teasel (*Dipsacus sylvestris*), a monocarpic perennial plant, using a projection interval of one year. (Note: a monocarpic plant flowers just once and then dies.)

Figure 1: Teasel

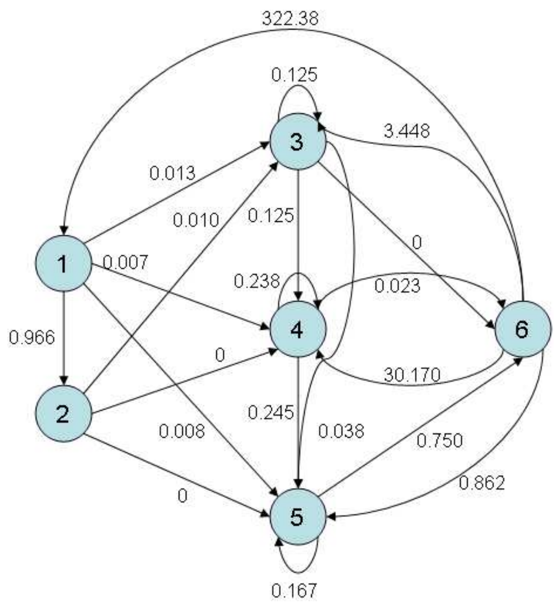
We consider six stages: 1, first-year dormant seeds; 2, second-year dormant seeds; 3, small rosettes; 4, medium rosettes; 5, large rosettes; and 6, flowering plants. The following life cycle graph describes the flow from stage to stage:

### Question One

Construct the transition matrix for teasel.

### Question Two

Assuming initial population densities of 63.95 first-year dormant seeds, 30.47 second-year dormant seeds, 0.87 small rosettes, 2.59 medium rosettes, 0.88 large rosettes, and 0.26 flowering plants, plot the stage distributions of the teasel present in time up to 5 years. Plot (a) on six different subplots; (b) on a single plot including a legend in your plot identifying the lines.




## Question Three

Plot the fraction of the total population at each time in each stage. What do you observe? You may need to do this for 20 years.