MATH495.3 (CRN 13695)

Laboratory # 5 : Continuous logistic map

Create a word document containing the MATLAB commands you used and the answers you obtained (including plots) for the following questions.

Definitions

The continuous-time logistic equation is

$$\left.\begin{aligned} \frac{dP}{dt} &= rP(1-\frac{P}{K}) \\ P(0) &= P_0 \end{aligned}\right\}, \tag{1}$$

where r = per capita growth rate and K = carrying capacity.

Non-dimensionalizing (1) by letting $x = P/K$, the equivalent non-dimensional logistic equation is

$$\left.\begin{aligned} \frac{dx}{dt} &= rx(1-x) \\ x(0) &= x_0 \end{aligned}\right\}. \tag{2}$$

Given the general scalar initial value problem

$$\left.\begin{aligned} \frac{dx}{dt} &= f(x), \\ x(0) &= x_0 \end{aligned}\right\}, \tag{3}$$

[a] $x^\star$ is called an *equilibrium point* if $f(x^\star) = 0$;

[b] an equilibrium $x^\star$ is *stable* if $f'(x^\star) < 0$ and *unstable* if $f'(x^\star) > 0$.

Problem 1

[a] Use `dfield8` to plot some solutions of the equation (2).

[b] Find the equilibria of equation (2) and determine their stability. Assume that $r > 0$.

Problem 2

Create the following MATLAB function

```
function [] = logistic(T,x0)
[t,x]=ode45(@logisticDE, [0,T], x0);
plot(t,x,'LineWidth',2)
title(['Logistic:  x(0) = ', num2str(x0)]);
xlabel('t')
ylabel('x')
end

function f = logisticDE(t,x)
r = 0.3
f = r*x*(1-x);
end
```

Plot solutions of for a range of initial conditions

The function logisticDE is called from inside another MATLAB *function, ode45. The parameter list for ode45 includes a* **function handle** *indicated by the @ symbol.*

Problem 3

To explore the role of the growth rate r we wish to pass this value as a parameter to the function which evaluates the rhs of the differential equation.

```
function [] = logistic2(T,r,x0)
[t,x]=ode45(@(t,x) logisticDE2(t,x,r), [0,T], x0);
plot(t,x,'LineWidth',2)
title(['Logistic:  r = ',num2str(r),', x(0) = ',num2str(x0)]);
xlabel('t')
ylabel('x')
end

function f = logisticDE2(t,x,r)
f = r*x*(1-x);
end
```

Plot the solution for several different values of r and several different initial conditions x_0. Describe how r affects the solution.