MATH495.3 (CRN 13695)

Laboratory # 4 : Discrete logistic map

Discrete logistic growth, given by the recursive equation

$$x_{n+1} = \lambda x_n(1 - x_n) \tag{1}$$

where $x_n \in [0,1]$ is the population *density* of the nth generation, is used to model population growth with a *carrying capacity*. Carrying capacity is the (equilibrium) number of organisms of a particular species that can be supported indefinitely in a given environment. For example, the total amount of food (plants, insects, smaller animals) and water in a particular animal's habitat influences the capacity of that habitat to support the life of more individuals.

- When $x_n \approx 0$, then $1 - x_n \approx 1$ and $x_{n+1} \approx \lambda x_n$, i.e. the population is growing (or decaying) at a constant rate.
- As x_n increases, the term $-\lambda x_n^2$ also increases in magnitude and "slows" the population growth.

We will consider the behavior of this model for specific values of λ. Please submit a word document with your m-file, the plots for each value of λ, and the answers to each question.

1. What is the carrying capacity (equilibrium size) of the model for $\lambda = 0.5, 1.5, 2.5$? You should be able to do this without any programming.
2. Start by creating a new m-file. You will first need to define the parameter λ. For each value of $\lambda = 0.5, 1.5, 2.5, 3.2, 3.5$, and 3.7, simulate the discrete logistic growth for 50 generations, given an initial population of 0.25.
3. Plot the dynamics for each value of λ (population versus generations) in a new figure window.
4. *Extra credit:* Plot the fixed point iteration for each of your choices of λ.
5. *Extra special credit:* How does the value of λ affect the population dynamics? Please give a detailed summary of this. If you need to plot the dynamics for more values of lambda, please do so.