MATH495.3 (CRN 13695)

Laboratory # 2 : Functions and simple flow control

1) Review of Arrays

The first 8 Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21. The first 8 triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36. Construct two arrays called fibonacci and triangle whose elements are the first 8 elements of these two sequences. Let F_i and T_i be the ith Fibonacci number and ith triangular number respectively.
Calculate the following

1. Compute $8 \times F_i$ for $i = 1, \ldots, 8$
2. Compute $F_i + T_i$ for $i = 1, \ldots, 8$
3. Compute $F_i \times T_i$ for $i = 1, \ldots, 8$
4. Compute $\frac{F_i}{T_i}$ for $i = 1, \ldots, 8$
5. Compute $\sin(F_i)$ for $i = 1, \ldots, 8$
6. Compute e^{T_i} for $i = 1, \ldots, 8$
7. Compute $\max(F_i^3, T_i^2)$ for $i = 1, \ldots, 8$

2) Fibonacci Computations

Fibonacci numbers are defined as:

i) f(1)=1

ii) f(2)=1

iii) f(n)=f(n-1)+f(n-2) for $n \geq 3$

Create a function with input n that outputs the nth Fibonacci number. Test your function with a few examples from the console. Turn in the function m-file to me so that I can test it too!

3) Intro to Discrete-Time Dynamical Systems

Suppose a population changes only at a discrete set of times. Of course we live in a continuous-time world, but there are many situations where this approximation is useful. For instance, while a population grows continuously, we may only be able to monitor the population once a year. Let x_t denote the population at time t, where t may be measured in hours, days, years, microseconds, etc... For this problem, let's suppose t is measured in years. So, the population after 4 years is represented by x_4.

Let x_{t+1} represent the population at time $t+1$, that is, x_{t+1} is the population one year after year t. A discrete-time dynamical system is an equation which describes how a variable (such as a population) changes in time, but only for a discrete set of times. For example, if a population doubles every year, it would be modeled by the discrete-time dynamical system:

$$x_{t+1} = 2x_t \,.$$

A more general discrete-time dynamical system can be written as follows:
New_Population = Growth_Rate * Previous_Population.

Suppose that instead of doubling every year (a growth rate of 2), the growth rate of a population changes at random every year. Specifically, there is a 50% chance that the annual growth rate is 0.86 and a 50% chance that the annual growth rate is 1.16.

Write an m-file (not necessarily a function) that simulates the population growth over 1000 years, if the initial population $x_0 = 100$. Use the MATLAB function rand($\cdot$) to "decide" whether the population grows or decays in any given year. Plot the population against time, labeling your axes and titling the graph. Turn in the plot and the m-file you used to create it.