M192: ASSIGNMENT ONE

Due: Thursday, August 30, 2007.

Question One

Construct a truth table for each statement.

(a) $p \Rightarrow \neg q$

(b) $[p \wedge (p \Rightarrow q)] \Rightarrow q$

(c) $[p \Rightarrow (q \wedge \neg q)] \Leftrightarrow \neg q$

Question Two

Use truth tables to verify that each of the following is a tautology.

(a) $(p \wedge q) \Leftrightarrow (q \wedge p)$

(b) $(p \vee q) \Leftrightarrow (q \vee p)$

(c) $[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$

(d) $[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$

(e) $[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$

(f) $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$

These are the commutative, associative and distributive laws respectively.

Question Three

Define a new seqential connective ∇, called *nor*, by the following truth table

p	q	$p\nabla q$
T	T	F
T	F	F
F	T	F
F	F	T

(a) Use a truth table to show that $p\nabla p$ is logically equivalent to $\neg p$.

(b) Complete a truth table for $(p\nabla p)\nabla(q\nabla q)$.

(c) Which of the basic connectives $(p \wedge q, p \vee q, p \Rightarrow q, p \Leftrightarrow q)$ is logically equivalent to $(p\nabla p)\nabla(q\nabla q)$?

Question Four

Rewrite each statement using $\exists, \forall, \ni$ and $\Rightarrow$ as appropriate.

(a) There exists a positive number x such that $x^2 = 5$.

(b) For every positive number M there is a positive number N such that $N < 1/M$.

(c) If $n \geq N$, then $|f_n(s) - f(x)| \leq 3$ for all x in A.

Question Five

A function f is increasing iff for every x and for every y, if $x \leq y$, then $f(x) \leq f(y)$.

(a) Rewrite this statement using $\exists, \forall, \ni$ and $\Rightarrow$ as appropriate.

(b) Write the negation of this statement using $\exists, \forall, \ni$ and $\Rightarrow$ as appropriate.