

Probability

Probability of an event = likelihood that event will occur

Applies to events in which some randomness is involved: outcome cannot be predicted ahead of time. Call such things “random experiments”.

What do we mean by “an event”?

Depends—could be rolling a 5 on one die, could be rolling a 5 on a blue die and a 3 on a red die, could be rolling any combination that adds up to 8 on two dice.

Sometimes there are many different ways that an event could occur.

Example: Rolling two dice

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Notice that we distinguish between dice (order matters). Could be two different dice or could be one die thrown twice in succession.

There are 5 different ways that one could roll a total of 8.

Each distinct combination that could arise is called an **outcome**.

An **event** is any chosen set of outcomes.

Example:

(5, 3) = an outcome

Rolling a total of 8 = an event.

Given an experiment, call the set of all possible outcomes the **sample space** for that experiment.

Example: $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \dots (6,5), (6,6)\}$

An event is a **subset of the sample space**.

Example: $E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

First, will figure out how to count the number of outcomes in the sample set. Then, will use this to find the probabilities of events.

Multiplication rule

If an experiment can be broken into two independent parts, the number of possible outcomes equals the number of outcomes for the first part times the number of outcomes for the second part.

Can expand this to any number of parts.

Example: tossing a coin three times

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Three parts, two choices in each part.

Size of the sample space = $n = 2 * 2 * 2 = 8$

Example: making a random coffee drink

Imagine a coffee shop where you are randomly assigned a latte using lists of choices from different categories. There are 3 sizes, 3 choices of milk, 10 flavors (including no flavor), and 2 choices of temperature.

How many choices?

$$n = 3 * 3 * 10 * 2 = 180$$



Sometimes we over count when using the multiplication rule and have to divide by the number of repeats

Example: ice cream in a bowl

Say there are 31 flavors of ice cream at a parlor. We want to order a dish with two scoops of different flavors. How many possibilities?

$$31 \times 30 = 930$$

But wait—this counts Chocolate, Strawberry as different than Strawberry, Chocolate. These two choices are really the same.

Every choice will be listed twice this way, so have to divide the number above by 2.

$$930 / 2 = 465 \text{ different choices}$$



What about for 3 scoops?

$$31 \times 30 \times 29 = 26970$$

For 3 flavors, how many different orders can we write them in?

$$3 \times 2 \times 1 = 6 \quad (\text{remember sequential coalitions?})$$

$$26970 / 6 = 4495 \text{ different choices.}$$

If all outcomes are equally likely, we can easily compute the probability of an event.

Probability of event = $\Pr(E) =$

$$\frac{\text{Number of outcomes leading to event}}{\text{Total number of possible outcomes}}$$

Total number of outcomes = size of sample space.

Example: rolling an 8

Recall: 5 outcomes leading to event

36 total outcomes

$$\Pr(\text{rolling an 8}) = 5/36$$

What about rolling a 3?

Rolling 9 or higher?

Example: particular latte

In our random coffee shop, with 3 sizes, 3 choices of milk, 10 flavors, and 2 temperatures, what is the probability of getting a small, whole milk, no flavor, hot latte (call this drink L)?

Number of outcomes leading to L = $1 * 1 * 1 * 1 = 1$

Total outcomes = 180

$\Pr(L) = 1/180$

What about any hot drink?

Outcomes leading to hot = $3 * 3 * 10 * 1 = 90$

Total possible outcomes = 180

$\Pr(\text{hot}) = 90/180 = 1/2$

We can also get this by ignoring all options but temperature, since we don't care about them, and looking at just temperature choices.

1 outcome leads to event (hot)

2 total possible outcomes (hot or cold)

$\Pr(\text{hot}) = 1/2$

Example: getting tails at least once in three coin flips

Getting tails exactly once in three coin flips