**Remember: The Brute Force Algorithm finds the weight of every Hamilton Circuit and chooses the cheapest one.**

**Pro: Guaranteed to find the most efficient circuit**

**Con: Can be a lot of work to carry out the algorithm**

**(each increase in vertices increases the work by a factor equal to the # of vertices in the graph)**

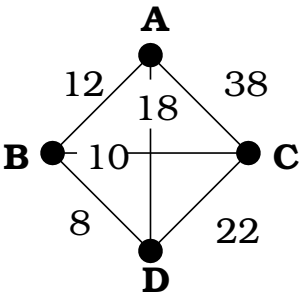
**Inefficient algorithm = an algorithm for which the number of steps needed to carry it out grows disproportionally with the size of the problem**

| vertices | # of circuits |
|---|---|
| 5 | 24 |
| 6 | 120 |
| 7 | 720 |
| 8 | 5040 |
| 9 | 362880 |
| 10 | 39916800 |

**Brute Force Algorithm is inefficient**

**Nearest Neighbor Algorithm- start at home and follow the cheapest choices from each vertex**

1. **Pick a vertex as the starting point**
2. **From the starting vertex, go to the vertex for which the corresponding edge has the smallest weight**
3. **Continue building the circuit, one vertex at a time.**
   - **Do not go to a vertex that has already been visited**
   - **Do not go to a vertex if it creates a small circuit**
4. **From the last vertex, return to the start.**




**Start at vertex A. What is the smallest number from A?**

**Go to vertex B, again, what is the smallest number from B?**

**Go to vertex D, what is the smallest number from D? (We can't go to A)**

**Only choice is C, and then back to A.**

**Circuit: ABDCA**

**Weight: 80**

**Is this the most efficient route?**

**Pro: Much less work!**

**Con: Doesn't necessarily produce the optimal Hamilton circuit (compromise answer)**

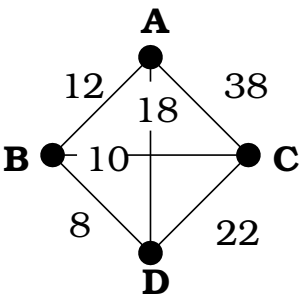
**Efficient Algorithm = an algorithm in which the number of steps needed to carry it out grows in proportion to the size of the input to the problem**

**Nearest Neighbor is efficient**

**Repetitive Nearest Neighbor Algorithm- Builds on Nearest Neighbor, but does the problem for each vertex.**

1. **Let X be any vertex. Apply the nearest neighbor algorithm using X as the starting vertex and calculate the total cost of the circuit obtained.**
2. **Repeat the process using each of the other vertices of the graph as the starting vertex.**
3. **Of the Hamilton circuits obtained, keep the best one.**

**We've already done the Nearest Neighbor starting with vertex A: weight = 80.**

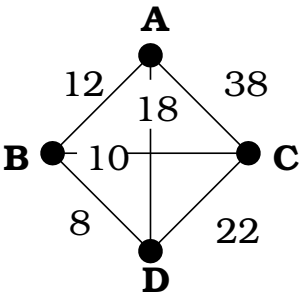


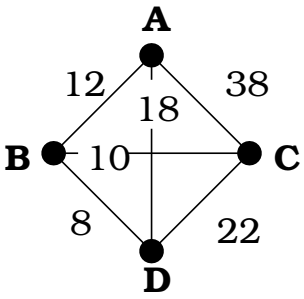

**Pick a different vertex, say B.**

**From B, the nearest neighbor is D, then A, and finally C.**

**Route: BDACB. Is this the same as ACBDA?**

**Weight: 74**




**Pick yet another vertex, C.**

**From C, nearest neighbor is B, then D and then A.**

**Route: CBDAC, same as ACBDA, weight 74.**

**Try starting at D:**

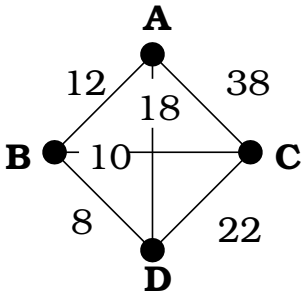
**From D, nearest neighbor is B, then C then A.**

**Route: DBCAD, which is the same as ADBCA, weight 74.**

**Best route found had a weight of 74. Is this the optimal route?**

## Cheapest Link Algorithm

1. **Pick the link with the smallest weight first. Mark the corresponding edge.**
2. **Pick the next cheapest link and mark the corresponding edge (note- This edge does not have to touch the edge already marked.)**
3. **Continue picking the cheapest link available and marking the corresponding edge except when:**
   (a) **It closes a circuit**
   (b) **It results in three edges coming out of a single vertex**
4. **When there are no more vertices to link, close the marked circuit.**




**Mark 8 first, then 10.**

**The next cheapest is 12, but picking AB gives us three lines coming out of B.**

**The next is 18, and we have now linked all vertices. Close the circuit by connecting A and C.**

**Route: ADBCA or ACBDA, weight 74.**

**Is this the optimal route?**

Approximate Algorithm- an algorithm that produces solutions that are most of the time reasonably close to the optimal solution.

Note: Last 3 methods are all Approximate Algorithms!

Try Nearest Neighbor, Repetitive Nearest Neighbor and Cheapest Link algorithms on the following examples: