Fair Division

Divide a set of objects in “fair” or “equal parts”

Fair Division is easy if all items have the same value: i.e. dividing a pizza between 4 people

Fair Division becomes harder when items have different values

Examples: inheritances, divorces, dividing prizes between several winners…

METHOD OF SEALED BIDS

A method of fair division

Works well when dividing fewer items between more people

STEP 1: THE BIDS
    Each player “bids” for each item
    Bids must be honest assessments of the value/worth (sealed bid)

STEP 2: THE ALLOCATION
    Each item goes to the highest bidder
STEP 3: THE PAYMENTS

Based on Step 2, each player pays in or is paid by the estate.

a. Calculate the fair share- Add up each player’s bids and then divide by the number of players
b. If the item allocated is greater than fair share, player pays the difference into the estate
c. If the item allocated is less than fair share, player receives the difference from the estate

STEP 4: DIVIDING THE SURPLUS

a. Divide the money left in the estate by the number of players
b. Give each player one share of the leftover money

NOTE:

1. Each player must have enough money to pay in (if necessary)
2. Each player must be willing to accept money vs. any item (nothing is priceless)
José and Betty are getting a divorce. The only thing worth splitting is the house.
Betty bids $142,000
José bids $130,000

Betty gets the house.

Fair shares:

Betty bids $142,000. There are 2 players, so in her opinion, each share is worth $142,000/2 or $71,000.

José bids $130,000, so in his opinion, each share is worth $130,000/2 or $65,000.

Betty gets the house at $142,000 and pays the estate $71,000.

José gets his $65,000 share from the estate, leaving a surplus of $6,000.

Split the surplus, $3,000 each.

End result:

Betty: Gets house, pays estate 68,000.
José: Gets $68,000 cash.

With 2 players and 1 item, you can pick the middle value of the two bids. The high bidder gets the item and pays the loser half of that value in cash.
Three heirs (Andre, Bea, and Chad) wish to divide up an estate consisting of a house, a small farm, and a painting, using the method of sealed bids.

Step 1: The Bids

<table>
<thead>
<tr>
<th></th>
<th>Andre</th>
<th>Bea</th>
<th>Chad</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>150,000</td>
<td>146,000</td>
<td>175,000</td>
</tr>
<tr>
<td>Farm</td>
<td>430,000</td>
<td>425,000</td>
<td>428,000</td>
</tr>
<tr>
<td>Painting</td>
<td>50,000</td>
<td>59,000</td>
<td>57,000</td>
</tr>
</tbody>
</table>

Step 2: The Allocation
Chad gets the house
Andre gets the farm
Bea gets the painting

Step 3: The Payments
Fair share: Andre = 210,000
           Bea = 210,000
           Chad = 220,000
Chad gets 45,000 from the estate
Andre pays the estate 220,000
Bea gets 151,000 from the estate

Step 4: Dividing the leftovers
Estate has 220,000, estate pays 196,000
Leftover: 24,000 (each player gets 8,000)
Method of Markers

Another method of fair division

Best for dividing a large number of items between a small number of people

Best if items are of similar value

(i.e. doesn’t work if dividing a sports car and candy)

Method:
- Line up the items

- Step 1: The Bids - Each player marks what they think is a fair division (i.e. if 4 players, each player divides the items into fourths)

- Step 2: The Allocation
  - Start at the left side of the line of items
  - Work your way to the right, stopping at the “first” marker
  - The first items go to the person with this “first” marker
  - Continue working to the right, stopping at the soonest “second” marker
  - The second player gets the items between his first and second markers
• Continue moving right, stopping at the soonest “third” marker
• The third player gets the items between her second and third markers
• Finally, the last person gets the items between the third marker and the end

- Step 3: Divide up leftovers (draw lots)

Guarantees that each player ends up with one of his bids

Is fair because the players made the divisions that they ended up with

Method of Markers Example 1

\[
\begin{align*}
&\text{C}_1 \quad \text{A}_1 \quad \text{B}_1 \\
&\text{C}_2 \quad \text{A}_2 \quad \text{B}_2
\end{align*}
\]

Leftovers: \(\text{truck}\) and \(\text{house}\)
Method of Markers Example 2

\[
\begin{array}{cccccccc}
\spadesuit & \heartsuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\heartsuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\end{array}
\]

\[\begin{array}{cccccccc}
C_1 B_1 & C_2 A_1 & A_2 & B_2 D_1 D_2 A_3 C_3 & B_3 \\
D_3 & \\
\end{array}\]

Leftovers
\[
\begin{array}{cccc}
\spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\end{array}
\]

Method of Markers Example 3

\[
\begin{array}{cccccccc}
\spadesuit & \heartsuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\heartsuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\end{array}
\]

\[\begin{array}{cccccccc}
D_1 A_1 & A_2 B_1 & A_3 & B_2 B_3 C_1 D_3 & C_2 C_3 \\
D_2 & \\
\end{array}\]

Leftovers
\[
\begin{array}{cccc}
\spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\spadesuit & \spadesuit & \spadesuit & \spadesuit \\
\end{array}
\]
Extra examples

Bob, Ann, and Jane wish to dissolve their partnership using the method of sealed bids. Bob bids $240,000 for the partnership, Ann bids $210,000, and Jane bids $225,000.

Step 1: Bids: Bob Ann Jane
        240,000 210,000 225,000

Step 2: Allocation: The business goes to Bob

Step 3: Payments:
Fair shares:
Bob = $80,000, owes estate $160,000
Ann = $70,000, paid by estate
Jane = $75,000, paid by estate

Step 4: Dividing the Surplus:
Estate received $160,000
Estate paid out $145,000 ($70,000 + $75,000)
Surplus = $15,000 ($160,000 - $145,000)
Divide it evenly among the 3.

Bottom line:
Bob gets business, pays out a total of $155,000
Anne gets $75,000
Jane gets $80,000
Method of Markers Example

In groups of 3 or 4, divide up the items below using the method of markers.

Items to divide:

phone         tv         coffee table
aquarium      playstation II cd player
Matrix DVD    house plant microwave
toaster       radio       dvd player
VCR           ant farm    laptop
blender       mini fridge guitar
Led Zeppelin box set(10 discs)
Grand Theft Auto: Vice City