M155 Exam 1 Review Sheet

1.1 – Biology and Dynamics

• Terminology

Discrete-Time Dynamical Systems *Probabilistic Dynamical Systems*
Continuous-Time Dynamical Systems

1.2 – Variables, Parameters, and Functions in Biology

• Terminology

Variable	*Argument (Input)*	*Domain*
Parameter	*Value (Output)*	*Range*
Relation	*Independent Variable*	*Commute*
Function	*Dependent Variable*	*Inverse*

• Identify relations and functions
    • Functions are relations where each input is associated with exactly one output
• Graph functions
• Evaluate functions using values and parameters
• Horizontal and vertical line test
    • A relation is not a function if a vertical line passes through the graph more than once
    • A function doesn't have an inverse if a horizontal line crosses a graph more than once
• Find inverses of functions which have them
• Combining functions (addition, multiplication, composition)

1.3 – The Units and Dimensions of Measurements and Functions

• Terminology

Dimensions	*Conversion Factors*	*Scaling*
Units	*Fundamental Relations*	*Shifting*

• Convert between units using conversion factors
• Translate between dimension using fundamental relations
• Compare two values with different units and dimensions
• Composing functions with dimensions
• Scaling functions
    • Multiply the output by a value to scale vertically
    • Multiply the input by a value to scale horizontally
• Shifting functions
    • Add a value to the output to shift vertically
    • Add a value to the input to shift horizontally

1.4 – Linear Functions and Their Graphs

• Terminology

Linear Function *Proportional Relation*
Interpolate *Slope*

• Recognize a linear function
• Find slope between two points
• Point-Slope Form for a line
    • $y = m(x - x_0) + y_0$

• Slope-Intercept Form
    • $y = mx + b$
• Find equation of a line from data (two points, slope and point, etc)
• Use a linear function to interpolate
• Plot line from its equation
• Find intersection of lines

1.5 – Discrete-Time Dynamical Systems

• Terminology

Discrete-Time Dynamical System *Updating Function*

• Understand the concept of updating functions
• Recognize updating functions
    • Example: Medication Concentration: $M_{t+1} = 0.5M_t + 1.0$
• Manipulate updating functions
    • Composition
    • Inverse
    • Convert units
    • Translate dimensions
• Find solution from updating function
    • Example: Updating Function: $b_{t+1} = 2.0b_t \quad \rightarrow \quad$ Solution: $b(t) = (2.0)^t b_0$
    • Example: Updating Function: $h_{t+1} = h_t + 1.0 \quad \rightarrow \quad$ Solution: $h(t) = h_0 + (1.0)t$

1.6 – Analysis of Discrete-Time Dynamical Systems

• Terminology

Cobwebbing *Equilibria*

• Know how to cobweb
• Find equilibria graphically and algebraically

1.7 – Expressing Solutions with Exponential Functions

• Terminology

Exponential Function *Natural Log* *Doubling Time/Half-Life*

• Per capita production (r) for $b_{t+1} = rb_t$
    • Population increases if $r > 1$
    • Population decreases if $r < 1$
    • Population remains constant if r = 1
• Know laws of exponents (page 81)
• Know laws of logarithms (page 83)
• Convert a solution to exponential form (page 84)
• For solution in exponential form $S(t) = S(0)e^{\alpha t}$
    • Population increases if $\alpha > 0$
    • Population decreases if $\alpha < 0$
    • Population remains constant if $\alpha = 0$
• Solve doubling-time, half-time, and other related problems
• Plot semilog graphs

1.8 – Oscillations and Trigonometry

• Terminology

Oscillations	*Amplitude*	*Period*
Cosine Function	*Minimum*	*Phase*
Average	*Maximum*	

• Convert between degrees and radians

• Find average, amplitude, phase, period, maximum, and period from cosine graph or function

• Graph cosine function from its function

1.9 – A Model of Gas Exchange in the Lung

• Terminology

Weighted Average *Ambient Concentration*

• Understand and use model of gas exchange

- $c_{t+1} = (1-q)c_t + q\gamma$ $q =$ fraction of air exchanged, $\gamma =$ ambient concentration

• Find weighted averages

• Find equilibria of lung model

• Understand and use lung model with absorption

- $c_{t+1} = (1-q)(1-\alpha)c_t + q\gamma$ $\alpha =$ fraction of chemical absorbed

1.10 – An Example of Nonlinear Dynamics

• Terminology

Mutant Type	*Stable Equilibria*
Wild Type	*Unstable Equilibria*

• Analyze updating function for fraction of mutant bacteria population

- $p_{t+1} = \dfrac{sp_t}{sp_t + r(1-p_t)}$ $s =$ mutant per capita production $r =$ wild per capita production
- Equilibria are always $p^* = 0$ and $p^* = 1$
- If $s > r$, mutant population will reach 100%, $p^* = 0$ is unstable and $p^* = 1$ is stable
- If $r > s$, mutant population will reach extinction, $p^* = 0$ is stable and $p^* = 1$ is unstable

• Recognize equilibria as stable (solutions move toward it) or unstable (solutions move away)

1.11 – Excitable Systems I: The Heart

• Terminology

2:1 AV Block *Wenckenbach Phenomenon*

• Understand and use model for heart potential

- $V_{t+1} = \begin{cases} cV_t + u & cV_t \le V_c \\ cV_t & cV_t > V_c \end{cases}$ where $c = e^{-\alpha\tau}$
- $c =$ potential decay, $V_c =$ threshold potential, $u =$ beat potential

• Identify whether healthy heart, 2:1 block, or Wenckenbach phenomenon from graph or function

• Find equilibria

- Equilibrium for healthy heart: $V^* = \dfrac{u}{1-c}$ only if $cV^* \le V_c$

2.1 – Introduction to Derivatives

• Terminology

Average Rate of Change *Secant Line*
Instantaneous Rate of Change *Tangent Line*
Base Point *Differential Notation*
Derivative *Prime Notation*
Limit

• Understand the difference between instantaneous and average rates of change
• Find average rates of change
• Graph and find equations of secant lines
• Graph and find equations of tangent lines
• Understand the instantaneous rate of change is called the derivative

2.2 Limits

• Terminology

Left-hand Limit *Right-hand Limit* *Infinite Limit*

• Compute limits
• Find instantaneous rate of change
    • $\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$
• Find how close an input must be for an output to be within a certain value

2.3 – Continuity

• Terminology

Continuous *Discontinuous*

• Recognize if a function is continuous or discontinuous
    • Continuous at the point x=a if $\lim_{x \to a} f(x) = f(a)$, else it is discontinuous
    • Continuous if continuous for all x in the domain

2.4 – Computing Derivatives: Linear and Quadratic Functions

• Terminology

Differentiable Functions *Critical Point* *Decreasing*
Differentiable *Increasing*

• Three ways a function can fail to be differentiable
    • Graph has a jump
    • Graph has a corner
    • Graph is vertical
• Determine critical points
• Find the derivative of linear function
• Find derivative of quadratic functions