Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have one hour and 45 minutes to work on the exam.

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1. (16 pts) Find the derivatives of the following functions. You do not have to simplify your answers. Be sure to use parentheses to indicate multiplication where appropriate.
   a) \( f(x) = x^9 e^{-3x^2-32} \).

   b) \( f(x) = \frac{11}{x^4 - 6x + 13} \).

   c) \( f(x) = 5 \ln(\cos x) \).

   d) \( f(x) = (\cos 2\pi x)(\sin \pi x) \).
2. (20 pts) Given the function \( f(x) = \frac{x^2}{2x^2 + 6x + 2} \) on the interval \((-\infty, \infty)\)

a) Find \( f_\infty(x) \), the leading behavior of \( f(x) \) as \( x \to \infty \), and \( f_0(x) \), the leading behavior of \( f(x) \) as \( x \to 0 \)

b) Use the method of matched leading behaviors to sketch a graph of \( f(x) \) on the interval \([0, \infty)\). Label your axes and indicate where you have graphed \( f_0(x) \) and \( f_\infty(x) \).
c) Find the intervals where $f(x)$ is increasing and decreasing on $(-\infty, \infty)$.

d) Find any local minima or local maxima of $f(x)$ on $(-\infty, \infty)$.

e) Do you think your graph from part b is reasonably accurate based on your answers to parts c and d? Why or why not?
3. (12 pts) A runner’s arm swings rhythmically according to the equation

\[ y(t) = \frac{\pi}{8} \cos[5\pi(t - 2/5)] \]

where \( t \) is time in sec, and \( y(t) \) is the angle between the actual position of the upper arm and the downward vertical position.

a. Calculate the velocity \( y'(t) \) and acceleration \( y''(t) \).

b. Give two times when the velocity is zero. What is happening at these times?

c. Is the function \( y(t) \) concave up or concave down when \( t = 5 \text{ s} \)? (Justify your answer) What does this tell you about the runner’s arm at this time?
4. (16 pts) Evaluate the following limits. Show all of your work. If you use L’Hospital’s Rule, justify why it can be applied each time you use it. If you use knowledge of leading behaviors, justify your work.

a) \( \lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 4} \)

b) \( \lim_{x \to 0} \frac{\tan x}{\sin x} \)

c) \( \lim_{x \to \infty} \frac{e^{-3x} + 4x}{x^2 + e^x} \)

d) \( \lim_{x \to \infty} \frac{\ln(2x + 13)}{\sqrt{x + 20}} \)
5. (12 pts) Suppose the amount of a certain chemical absorbed by the lung is

\[ A(c) = 0.002ce^{-\alpha c}, \quad \alpha > 0. \]

a) Find the concentration \( c \geq 0 \) of chemical for which the amount absorbed is maximal. Justify that you have found the maximum.

b) What is the amount absorbed for very high concentrations? (Find \( \lim_{c \to \infty} A(c) \).)

Hint: \( A(c) = \frac{0.002c}{e^{\alpha c}} \). Do you think this amount is realistic? Why or why not?
6. (12 pts) The following updating function describes a crowded population $N_n$. 

$$N_{n+1} = \frac{\lambda N_n}{1 + aN_n}, \quad a, \lambda > 0$$

a) Find all equilibria of the updating function.

b) Use the stability test to determine under what conditions each equilibrium is stable.
7. (12 pts) Consider the function \( f(x) \) on the interval \([-3, 3]\) where
\[
f(x) = x^3 - 12x
\]
a) Find the critical points of \( f(x) \) on \([-3, 3]\).

b) Classify each critical point on \([-3, 3]\) as a local minimum, local maximum or neither. Justify your answer by using either the first or second derivative test.

c) Find the global maximum and global minimum on \([-3, 3]\) and give both the \( x \) and \( y \) coordinates of each.