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Exploiting the Colors of Soap Bubbles

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Abstract. Soap films and soap bubbles display properties relevant to both differential geometry and optics. We explore a method in which knowledge of the shape of a bubble and colors from interference patterns on the soap film can be used to determine the thickness of a bubble at a point on the surface, to try and determine the thickness when a bubble pops. We use a bubble stretched between a wire loop and a petri dish creating a catenoid shape. Using the colors on the surface of the bubble, the thickness can be determined. While thickness was determined for small sections of the bubble, the thickness of the bubble when it popped was unable to be determined for reasons explained. Additionally, a question about the pinch off of this system is explored.

Keywords: Bubbles, Soap Films, Interference

1 Introduction

Anyone who has played with soap bubbles as a child, or even has washed dishes at any point of their lives, has seen the beautiful colors created on the surface of soap bubbles. While beautiful, these bubble can serve a useful purpose in discovering properties of a bubble, like the thickness. In this paper, I examine a bubble stretched effectively between two loops. In this case, it is a petri dish and a wire loop. The purpose of my project was to determine if a bubble will pop after the soap film reaches a critical thickness.

This paper is organized as follows: In Section 2, we introduce the setup of our experiment. In Section 3, we show how colors on soap bubbles can be used to determing the thickness of the soap film. Section 4 describes the manner in which data was collected. Section 5 examines the results, and finally, Section 6 concludes and discusses future work.

2 Setup

The object under study was a soap film stretched between two circles. In my case, I used a wire loop with a diamter of 4.7 cm and a petri dish with a diameter of 5.5 cm full of bubble solution, as shown in Fig. 1.

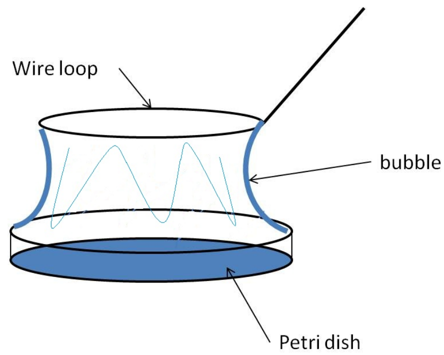


Figure 1: Setup of experiment. Wire loop forms a bubble with a petri dish.

To perform the experiment, we set up the bubble, LED, and camera as shown in Fig. 2.

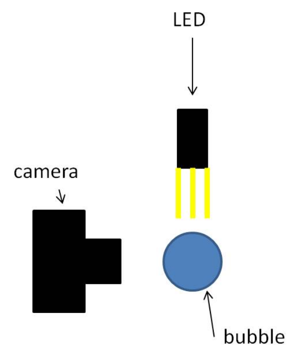


Figure 2: Top view of experimental setup.

Using a stand, we propped the wire loop up at varying heights of 1 cm, 2 cm, and 3 cm above the rim of the petri dish. We created the bubble by bringing the petri dish up to the loop and pulling down. We then let the bubble sit and filmed the changing colors until the bubble popped.

3 Theory

The colors on a soap film are due to the constructive and destructive interference of different wavelengths of light. Different wavelengths of light correspond to different colors. Using a combination of the wavenumber, $k = 2\pi/\lambda$ and Snell's Law, $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ we can find an expression for the thickness of the soap film for a given color and a given angle of incidence of incoming light. In the equation for Snell's Law, n_1 is the index of refraction of air (equal to 1), and n_2 is the index of refraction of the soap film, which is approximately 1.33 [3]. θ_1 is the angle of incidence of the incoming light measured from the normal of the surface, and θ_2 is the angle of incidence of the light refracted in the soap film as measured from normal to the surface. This is diagramed in Fig. 3.

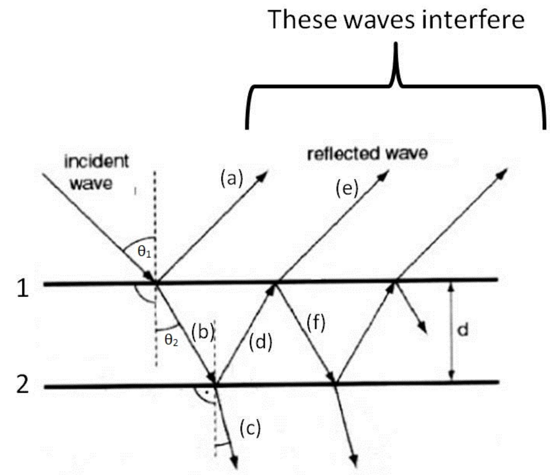


Figure 3: How light is transmitted and reflects through a soap film. Incident wave is either reflected (a) or transmitted (b) at interface 1. The transmitted (b) wave is either transmitted (c) or reflected (d) at interface 2. The reflected wave (d) is either transmitted (e) or reflected (f) at interface 1.

If (a) and (e) and the additional reflected waves in Fig. 3 are in phase, they will constructively interfere, creating a bright spot of that particular color. To find the phase difference between light reflecting off the top surface of the film and light reflected from the buttom surface of the film, we have the equation

$$\delta = k2n_2d\cos(\theta_2),$$

involving the fact that light that reflects off an object with a higher index of refraction than the light is traveling in undergoes a 180 phase change after reflection.

Next, we know by Fresnel's equations[1] that

$$\delta = \frac{4\pi d}{\lambda}\sqrt{n^2 - \sin^2(\theta_i)}.$$

In our case, we want δ to be equal to a multiple 2π for constructive interference. Using this fact and solving for d, we find

$$d = \frac{m\lambda}{2\sqrt{n_2^2 - sin^2(\theta_i)}}, \tag{1}$$

where m is a positive integer greater than zero. We need to know the wavelength and the incident angle. We will first examine the latter. Assuming our light source is parallel (Fig. 4), the tricky part is finding $\hat{n}$, the unit normal vector to the surface. This involves knowing a little more about the behavior of bubbles.

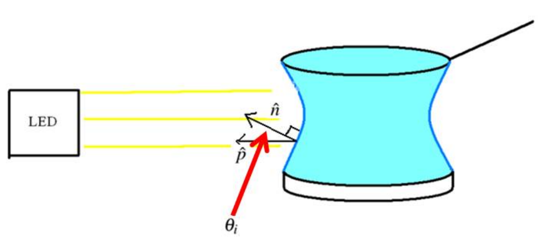


Figure 4: $\hat{n}$ is the unit normal vector to the surface, and $\hat{p}$ is the vector antiparallel to the incoming light.

Bubbles always assume the shape with minimal surface area. Having been studied previously[2], this minimum surface area shape is known to be a catenoid (Fig. 5).

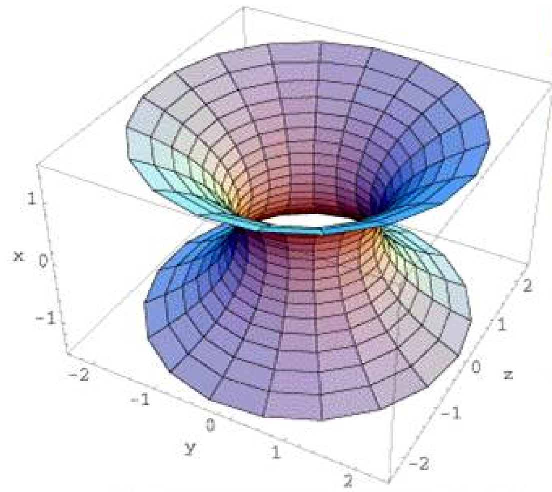


Figure 5: Shape of a bubble stretched between two loops. The catenoid minimizes surface area, or has a mean curvature of zero.

The surface is parameterized by

$$\vec{x} = x\hat{i} + \cosh(x)\cos(\theta)\hat{j} + \cosh(x)\sin(\theta)\hat{k},$$

where x is the distance from the center of the catenoid along the axis of rotational symmetry, and θ is the angle in the y-z plane as measured from the y-axis. Using the formula to find the angle between two vectors,

$$\theta_i = \cos^{-1} \frac{\hat{n} \cdot \hat{p}}{|\hat{n}||\hat{p}|},$$

and setting $\hat{p}$ to be along the y-axis, we find

$$\theta_i = \cos^{-1} \left(\frac{-\cos(\theta)}{\cosh(x)} \right).$$

We can then use this in our formula for d (Eq. 1). Now the only missing variable in this equation is the wavelength. This we can extract from the data we collect.

4 Data Collection

We used the color of the bubble to find the wavelength, and therefore, the thickness at any given point. We saw repeated bands of color during the experiment (Fig. 6).

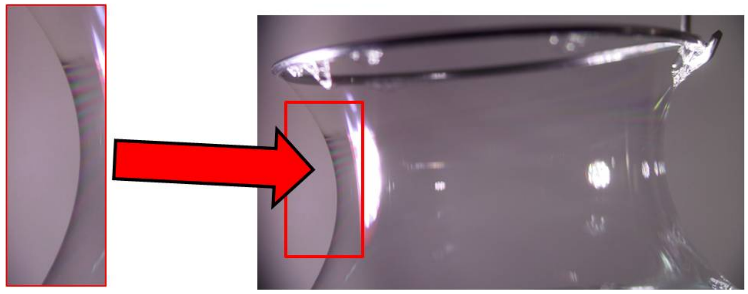


Figure 6: Typical data picture showing repeated bands of color. Note the colorless area above the visible bands of color, where the bubble becomes thinner than half the wavelength of visible light and thickness at this point can no longer be determined.

Based on the fact that colors can deceptively mix, the colors we pinpointed were those which are strongest in LED's, that of about 550 nm (Fig. 7).

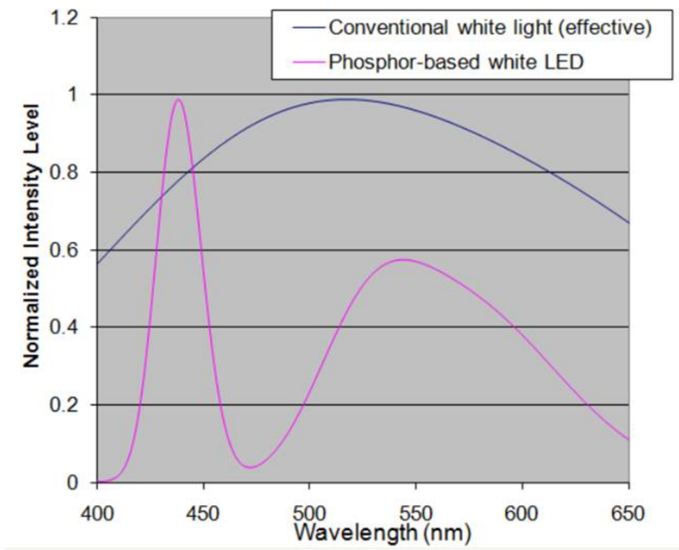


Figure 7: Sprectra of a typical white LED.

When collecting data, we had to wait until the color bands appeared. While the film was still rather thick, the colors of the soap film could not be observed. Once the strong color bands appeared, there was a colorless area above these colors which was too small to allow interference of the wavelengths in the visible spectrum (Fig. 6). Now, the bands of the same color appeared because the thickness of the soap film was an integer multiple of the wavelength of that particular color. Thus, after the first color band, each calculated thickness had to be multiplied by an increasing integer, as given by the m in Eq. 1.

5 Results

The final results were inconclusive. Fig. 10 shows the thinnest point of the bubble that could be identified for three different wire loop-petri dish separations. No correlation between when the bubble pops and the distance of the thinnest point visible from the center of the bubble was found.

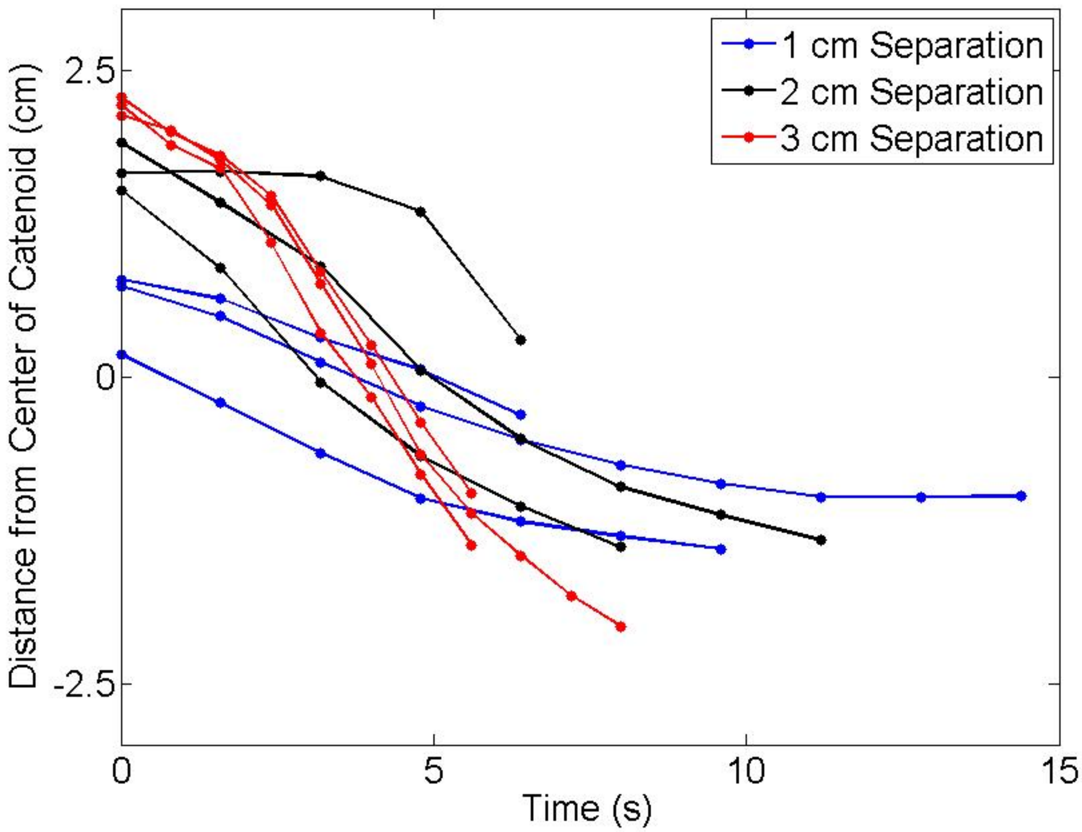
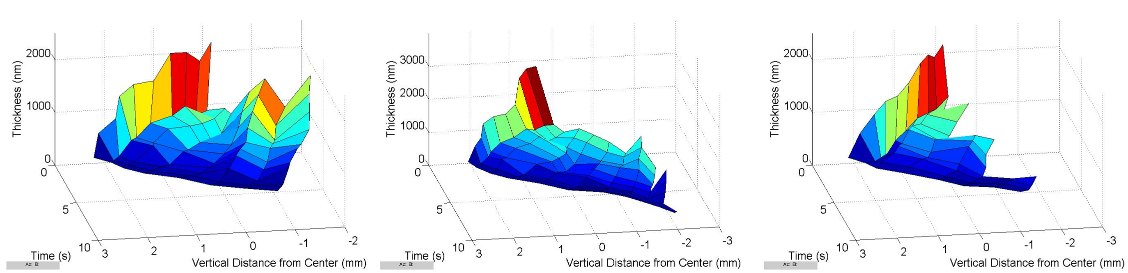


Figure 8: Distance from the center of the catenoid versus time from when the colorless section of bubble first appeared near the wire loop. Each different line is a different trial at varying separations. The line stops when the bubble pops, which doesn't appear to correlate with any particular time for the given data.

Though we couldn't determine how thin the bubble was when it popped, we tried to interpolate from the graphs if the thickness of the bubble fell off linearly by distance from the center, but from the shown examples (Fig. 9), no correlation between the distance and thickness could be determined from the given information.



(a) Trial 1 (b) Trial 2 (c) Trial 3

Figure 9: Thickness profiles with time for the 3 cm separation of the wire and petri dish. Not enough data, given the small data range, was collected to determine if a trend in thickness profiles was present.

6 Conclusion

In conclusion, we were able to use interferometry to determine the thickness of the bubble for a small range of the bubble. For thickness smaller than half the wavelength of the smallest wavelength visible light, we could not determine the thickness of the bubble. Beyond about five wavelengths thick, we couldn't determine the thickness as clear colors were not visible. Given a smaller wavelength of light, we could determine the thickness of the bubble before it pops. The difficulty comes from the fact that we could no longer observe this smaller wavelength, as it would be outside the visible spectrum.

There were sources of error in this experiment. First of all, the bubble was not isolated from air currents, which can affect when a bubble pops. Also, the LED wasn't truly parallel. The wire loop wasn't perfectly circular, so the catenary shape may not have been precise. Future work would include possibly using some sort of camera that could pick up wavelengths outside the visible spectrum in order to observe thinner sections of the bubble.

6.1 Future Work

One area we explored but were limited by the speed of the video camera was in the pinch off. The pinch off occured when the wire loop was separated far enough from the petri dish that the waist of the catenoid touched and broke into two bubbles. Fig. 10 shows the expected behavior of the bubble when pulled away from the petri dish, as well as the observed behavior.

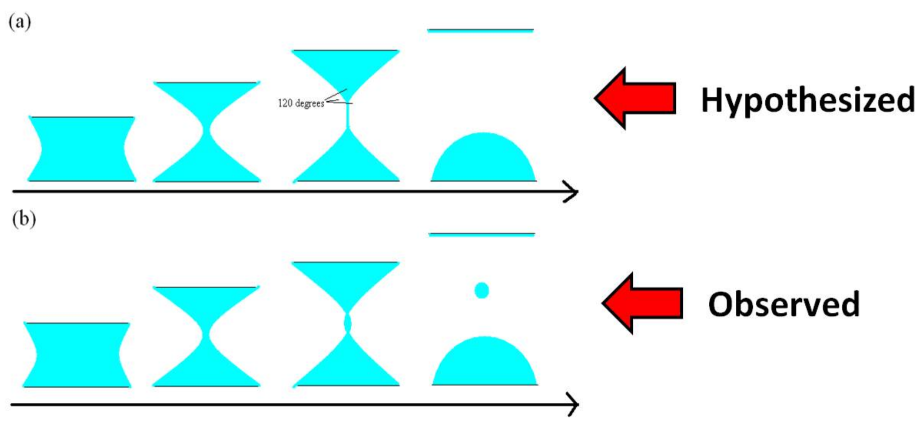


Figure 10: Expected and observed pinch off for the wire loop and petri dish setup. Shown in (a), we expected the waist of the canenoid to get small, touch, and the two separate bubbles form, one in the loop and one over the petri dish. (b) shows what we actually observed, in which air was trapped in the decreasing waist, creating a small bubble.

The third image from the left in (b) of Fig. 10 is shown in a picture in Fig. 11.

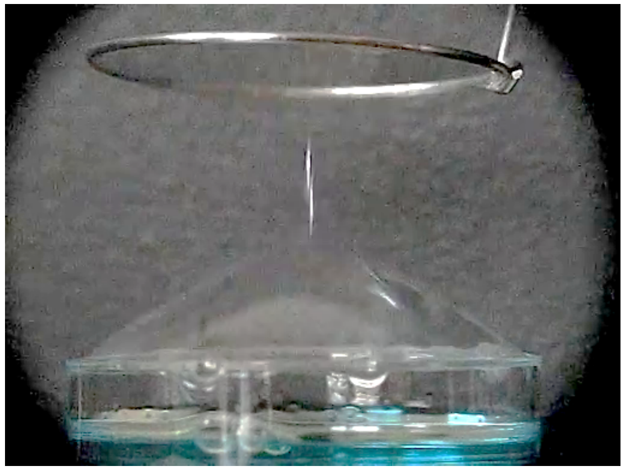
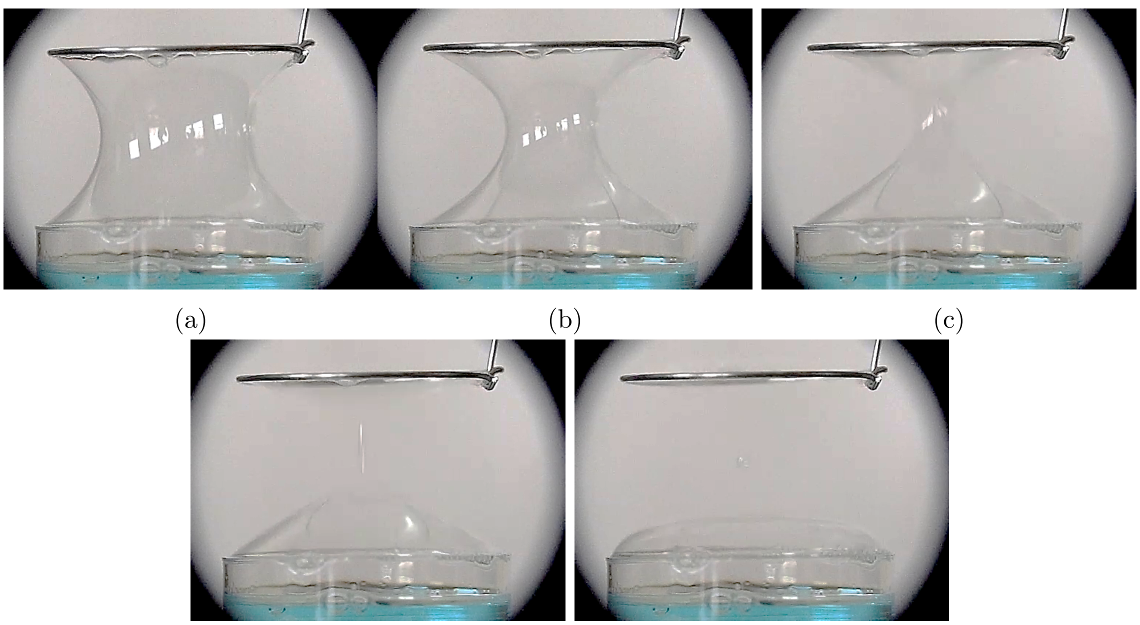


Figure 11: Unexpected behavior during the pinch off of the bubble. Air is trapped in the neck of the shrinking waist of the catenoid, which will separate into a small bubble.

A sequence of what occurs is shown in Fig. 12. The images appear blurry because of the speed of the pinch off.



(a) (b) (c)

(d) (e)

Figure 12: Time lapse of a bubble pinch off. (a) The waist shrinks. (b) Note that the entire curvature of the surface is convex. (c) Note that the curvature of the bubble half closest to the petri dish has changed from convex to concave. (d)Air is trapped in the neck of the catenoid. The curvature of the upper bubble is convex, while the curvature of the lower bubble is both convex and concave. (e)A small bubble appears after the catenoid has separated.

Possibly what is creating the unexpected bubble is that the air trapped in the catenoid neck on the loop side can escape, while the air on the petri dish side cannot escape. While the bubble membrane is normally strong enough to overcome any pressure differences and maintain its shape, in this case, the air pressure becomes too much. As can be seen in figure (c) of Fig. 12, the petri dish side of the catenoid begins to lose the convexity of its sides. The curvature changes, possibly due to larger air pressure. This could change the shape of the bubble enough to affect how the bubble pinches off.

While this could be a plausable explanation, when a bubble was created between two wire loops, occasionally a small bubble would appear. This suggests that the pressure idea could be incorrect. Another thought is that the fast moving air creates a low pressure area on either side of the neck, while the still air in the middle does not ”suck” in the bubble as tightly. This would be accurate, except for the fact that the lowest pressure would be in the center, where the air is leaving, so the neck should pinch off the fastest in this region.

One other possibility lays in the Rayleigh-Plateau Instability. This instability describes an instability which occurs on a cylindrical column of fluid. When the column length becomes greater than the column diameter by a factor of about 3.13, sinusoidal variations occur in the surface due to pressure differences and surface tension[4]. This would need to be looked into further to determine if a Rayleigh-Plateau Instability is applicable to our system. Thus, this pinch-off phenomena remains a mystery.

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