MATH 670 Intro to Manifolds : Exercise Sheet Nine

1. Let $f : S^n \to S^n$ be a map with degree not equal to $(-1)^{n+1}$. Prove that f has a fixed point.

2. Let $f : S^n \to S^n$ be a map with odd degree. Prove that there exists a pair of antipodal points $\{x, -x\}$ which are carried to antipodal points by f, i.e.,

$$\{f(x), f(-x)\} = \{y, -y\}$$

for some $y \in S^n$.