MATH 670 Intro to Manifolds : Exercise Sheet Eight

1. Let M be a connected manifold of dimension at least two and let $x_1, \ldots, x_k, y_1, \ldots, y_k$ be distinct points on M. Prove there exists a diffeomorphism $\phi : M \to M$ with $\phi(x_i) = y_i$ for all $i = 1, \ldots, k$.

2. Suppose $m < n$ and let f and g be any two embeddings $\mathrm{R}^m \hookrightarrow \mathbb{R}^n$. Prove that f and g are isotopic.

3. Let $f : (0, 1) \to \mathbb{R}^2$ be the embedding which takes t to $(t, 0)$, i.e., $(0, 1)$ is embedded along the x-axis. Find an isotopy of f which cannot be embedded in a diffeotopy of $\mathbb{R}^2$.

4. Let f and g be two maps from M to the unit sphere S^n such that for all $p \in M$, $f(p)$ and $g(p)$ are *not* antipodal points. Prove that f is homotopic to g.

5. Suppose $m < n$. Prove that every map $M^m \to S^n$ is homotopic to a constant map.