MATH 670 Intro to Manifolds : Exercise Sheet Seven

1. a) Let $M = \mathbb{R}$ and let v be the vector field which takes $x \in M$ to $x^2 \in T_xM$. Find the maximal solution curve through each point $x_0 \in M$. Describe and sketch the open set $A \subset \mathbb{R} \times M$ on which the maximal local flow of v is defined.

   b) Repeat part a) for the vector field $x \mapsto x^2 + 1$.

2. a) Let $G \subset \mathbb{R}$ be a closed subset and subgroup of $(\mathbb{R}, +)$. Show that either G is trivial, $G \cong \mathbb{Z}$, or $G = \mathbb{R}$.

   b) Let $\alpha_x : \mathbb{R} \to M$ be a flow line of a global flow. Show that $G := \{t \in \mathbb{R} | \alpha_x(t) = x\}$ is a closed subgroup of $(\mathbb{R}, +)$.

   c) What do parts a) and b) tell us about the behaviour of flow lines?

3. Let M be a manifold admitting partitions of unity. Show that every submanifold of M diffeomorphic to S^1 is the orbit of some global flow on M.

4. Find an injective immersion $\mathbb{R} \hookrightarrow \mathbb{R}^2$ whose image is not a flow line of a flow on $\mathbb{R}^2$.