

MATH 670 Intro to Manifolds : Exercise Sheet Six

1. Let M be a compact smooth manifold of dimension n and $f : M \rightarrow \mathbb{R}^{n+1}$ a smooth map such that the image $f(M)$ does not contain the origin. Show that there exists a line through the origin which meets $f(M)$ in only a finite number of points (possibly zero points).

Hint: Construct a map $M \rightarrow \mathbb{RP}^n$, then apply Sard's Theorem to find a regular value.

2. Construct an injective smooth map $f : S^1 \rightarrow \mathbb{R}^2$ whose image is the square

$$\{x \in \mathbb{R}^2 \mid \max\{|x_1|, |x_2|\} = 1\}.$$

Can f be an immersion?

Beware: The map $f = (f_1, f_2)$ must have continuous derivatives of all orders.