MATH 670 Intro to Manifolds : Exercise Sheet Five

1. *Milnor manifolds:* Let $\mathbb{RP}^n$ denote real projective space and define

$$H(m,n) := \{(x,y) \in \mathbb{RP}^m \times \mathbb{RP}^n | \sum_{i=0}^{m} x_i y_i = 0\}$$

for $m \leq n$, where $x = [x_0, \ldots, x_m]$ and $y = [y_0, \ldots, y_n]$ are homogeneous coordinates. Show that $H(m,n)$ is a smooth manifold of dimension $(m+n-1)$.

Hint: Write the equation in local coordinates, for example, on a coordinate patch

$$U_0 \times V_0 = \{(x,y) | x_0 \neq 0, y_0 \neq 0\}.$$

2. *Brieskorn manifolds:* For an integer $d \geq 0$, let $W^{2n-1}(d)$ be the set of points $(z_0, \ldots, z_n)$ in $\mathbb{C}^{n+1}$ such that

$$z_0^d + z_1^2 + z_2^2 + \ldots + z_n^2 = 0$$

and

$$z_0 \bar{z}_0 + z_1 \bar{z}_1 + \ldots + z_n \bar{z}_n = 2.$$

Then $W^{2n-1}(d)$ is a smooth manifold of dimension $2n-1$. Prove this for $W^3(2)$.

If you are feeling ambitious, you could try to prove the statement for general d and n.