

MATH 670 Intro to Manifolds : Exercise Sheet Three

1. Let E be an arbitrary vector bundle. Show that $E \oplus E$ is orientable.
2. Show that $\mathbb{R}P^n$ is orientable for odd values of n and non-orientable for even values of n .
3. Show that if $M \times N$ is orientable, then both M and N must be orientable.
4. Let L be a line bundle on a manifold X , and assume that X admits partitions of unity subordinate to any open cover. Show that $L \otimes L$ is trivial.
5. Show that if E is a trivial rank n bundle on X with a Riemannian metric, then there is a bundle isomorphism $E \cong X \times \mathbb{R}^n$ which is an isometry on each fibre.