Name: Time allowed: 50 minutes.
Calculators are not allowed.

M369 Linear Algebra (section 3) : Practise for 1st Midterm Exam

Problem	1	2	3	4	5	6	Extra Credit	Total
Score								
Maximum	12	15	18	20	15	20	(15)	100

1. Determine whether the following subsets of vectors in R^3 are vector subspaces.

a) all vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ such that $x_1 - x_2 + x_3 = 0$;

b) all vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ such that $x_1 x_2 x_3 = 0$.

2. Solve the system of equations

$$\begin{aligned} x_1 + 3x_2 + x_3 + 2x_4 &= 0 \\ 2x_1 + 7x_2 + 3x_3 + 5x_4 &= 0 \\ 3x_1 + 7x_2 + x_3 + 4x_4 &= 0 \end{aligned}$$

and write down a basis for the solution space.

3. For each of the following sets of vectors in $\mathbb{R}^3$, determine whether they are linearly independent, whether they span $\mathbb{R}^3$, and whether they form a basis for $\mathbb{R}^3$.

a) $[\mathbf{v}_1, \mathbf{v}_2] = \left[\begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ 1 \\ 4 \end{pmatrix} \right]$;

b) $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = \left[\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right]$;

c) $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4] = \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right]$.

4. Let A be an $m \times n$ matrix.

a) Define the rank and the nullity of A, and state the Rank-Nullity Theorem.

b) Prove that if the rank of A is less than n, then there exists a non-zero vector $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{0}$.

5. Determine which of the following are linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^2$.

a) $L(\mathbf{x}) = \begin{pmatrix} 0 \\ x_1 + x_2^2 + x_3^3 \end{pmatrix}$;

b) $L(\mathbf{x}) = \begin{pmatrix} 2x_2 \\ (x_1+1)^2 - (x_1-1)^2 \end{pmatrix}$.

[Hint: First expand and simplify the right hand side.]

6. Let L be the linear operator on $\mathbb{R}^2$ given by

$$L(\mathbf{x}) = \begin{pmatrix} x_1 - x_2 \\ x_2 \end{pmatrix}.$$

a) Write down the matrix A representing L with respect to the standard basis for $\mathbb{R}^2$.

b) Write down the matrix B representing L with respect to the basis

$$F = [\mathbf{u}_1, \mathbf{u}_2] = \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 3 \end{pmatrix} \right].$$

[EXTRA CREDIT]

7. a) Find the transition matrix corresponding to the change of basis from

$$[\mathbf{u}_1, \mathbf{u}_2] = \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right] \qquad \text{to} \qquad [\mathbf{v}_1, \mathbf{v}_2] = \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right].$$

b) If $\mathbf{x} = 3\mathbf{u}_1 - \mathbf{u}_2$, find the coordinates of $\mathbf{x}$ with respect to $[\mathbf{v}_1, \mathbf{v}_2]$.