

Name:

Time allowed: 50 minutes.
Calculators are not allowed.

M369 Linear Algebra (section 1) : Practise for 1st Midterm Exam

1. Solve the system of equations

$$\begin{aligned} -3x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 0 \\ x_1 - 3x_2 + 2x_3 &= 0 \end{aligned}$$

and write down a basis for the solution space.

2. Let A be the 3×3 matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}.$$

- Write down the reduced row echelon form U of A .
- Use U to write down a basis for the row space of A .
- Use U to write down a basis for the column space of A .

3. Determine whether the following subsets of vectors in \mathbb{R}^2 are vector subspaces.

- all vectors $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ such that $x_1 + x_2 = 0$;
- all vectors $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ such that $x_1 - x_2 = 0$;
- all vectors $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ such that $x_1^2 - x_2^2 = 0$.

4. Determine which of the following are linear transformations.

- $L : \mathbb{R}^2 \rightarrow \mathbb{R}^1$ defined by $L(\mathbf{x}) = x_1x_2$;
- $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $L(\mathbf{x}) = \begin{pmatrix} x_1 + 2x_2 + 3x_3 \\ 4x_1 + 5x_2 + 6 \end{pmatrix}$.

5. Write down the transition matrix for the change of basis from $[2x - 1, 1 - 3x]$ to $[2, 1 - x]$ in P_2 .

6. For each of the following sets of vectors determine whether they are linearly independent, whether they span, and whether they form a basis for the given vector space.

a) $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = \left[\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right]$ in \mathbb{R}^3 ;

b) $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = \left[\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right]$ in \mathbb{R}^4 ;

c) $[\mathbf{v}_1, \mathbf{v}_2] = \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right]$ in \mathbb{R}^2 .

7. Let A be an $m \times n$ matrix whose columns are linearly dependent. Suppose that \mathbf{b} belongs to the column space of A . Prove that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

8. Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation defined by

$$L(\mathbf{x}) = x_2\mathbf{b}_1 + (x_1 + x_2)\mathbf{b}_2 + x_1\mathbf{b}_3$$

where $F = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$ is the basis

$$\mathbf{b}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{b}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{b}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

of \mathbb{R}^3 .

a) Find the matrix A which represents L with respect to the basis $E = [\mathbf{u}_1, \mathbf{u}_2] = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right]$ of \mathbb{R}^2 and the basis F of \mathbb{R}^3 .

b) Using the matrix A , find the coordinates of $L(\mathbf{u}_1 + \mathbf{u}_2)$ with respect to the basis F .