Mar 9

Note Title 3/9/2009

[65 - 75] B

#2 $\int_C y^3\,dx + (3x - x^3)\,dy$

$\overset{GT}{=} \iint_R (3 - 3(x^2+y^2))\,dA$

>

Curve is circle
where $3-3(x^2+y^2)=0$
$=$ unit circle

want to maximize

$$\iint_R 3-3(x^2+y^2)\,dA$$

$$f(x,y) = 3-3(x^2+y^2)$$

$$D^+ = \{(x,y) \mid f(x,y) > 0\}$$

$$D^0 = \{(x,y) \mid f(x,y) = 0\}$$

$$D^- = \{(x,y) \mid f(x,y) < 0\}$$

$$\iint_R f(x,y)\, dA$$

$$= \iint_{R \cap D^+} f(x,y)\, dA + \iint_{R \cap D^-} f(x,y)\, dA$$

$$\leq \iint_{R \cap D^+} f(x,y)\, dA$$

$$\leq \iint_{D^+} f(x,y)\, dA$$

So Int. is max on

<u>D^+</u>.

curve = ∂D^+

#3)

(r_n, θ_n) - polar

$r_n \searrow 0$

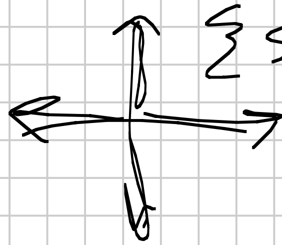
$\theta_n = \frac{2n\pi}{4}$

$\sum \vec{v}_n$ conv.

Show each coord. converges (cartesian)

$\vec{v}_n = (r_n \cos \theta_n, r_n \sin \theta_n)$

$\left\{\cos\left(\frac{2n\pi}{4}\right)\right\} = 0, -1, 0, 1, 0 \ldots$



$\left\{\sin\left(\frac{2n\pi}{4}\right)\right\}$

$= 1, 0, -1, 0, 1, 0 \ldots$

So both

$r_n \cos\frac{2\pi n}{4} + r_n \sin\frac{2\pi n}{4}$

are terms of alt. series

Suppose I looked at series

$$\sum_{n=1}^{\infty} r_n \cos(2\pi n \alpha)$$

$\alpha \in [0,1)$

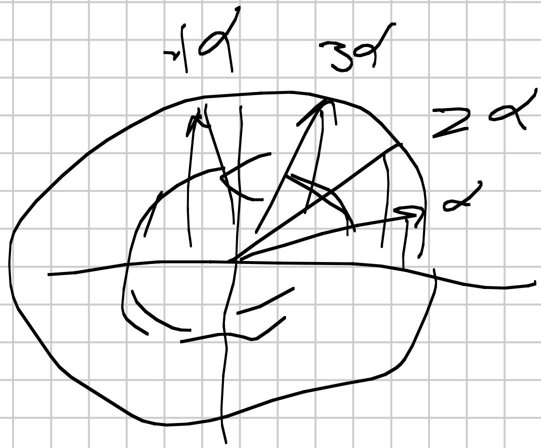
& $r_n \geq 0$ & decrease monotonically to 0,

Does this converge?

If $\alpha = 0$, maybe not.

If $\alpha = \frac{1}{4}$ - yes.

$\alpha = \frac{\sqrt{2}}{2}$.



Show: if $\alpha \neq 0$, sum converges.

$\cos(2\pi n\alpha)$

Can I show that the partial sums

$$\sum_{n=1}^{N} \cos(2\pi n\alpha)$$

stay bounded?

$$e^{2\pi i\alpha} = \cos 2\pi\alpha + i\sin 2\pi\alpha$$

So $\cos(2\pi n\alpha)$

$$= re\left(e^{2\pi i n\alpha}\right)$$

$$\sum_{n=1}^{N} \mathrm{re}\left(e^{2\pi i n\alpha}\right)$$

$$= \mathrm{re}\left(\sum_{n=1}^{N} e^{2\pi i n\alpha}\right)$$

$$\left(e^{2\pi i \alpha}\right)^n$$

$$= \left|\mathrm{re}\left(\frac{e^{2\pi i\alpha} - e^{2\pi i(N+1)\alpha}}{\underbrace{e^{2\pi i\alpha} - 1}_{\neq 0}}\right)\right|$$

$x_1 \quad \cdots \quad x_N$

$x_2 \quad \cdots \quad x_{N+1}$

$$\leq \left| \frac{e^{2\pi i \alpha} - e^{2\pi i (N+1)\alpha}}{e^{2\pi i \alpha} - 1} \right|$$

$$\leq \frac{2}{|e^{2\pi i \alpha} - 1|} = B$$

& By Dirichlet this series converges

Chapt 7

Sequences + Series of functions

$f_k : \underline{\underline{S}} \to \mathbb{R}$

Common Domain -

Consider $f_k \to f$

+ when $\sum f_k$ conv.

$f_k \to f$ -

at base you mean

for each $x \in S$,

$f_k(x) \to f(x)$

Defn If for all $x \in S$,

$f_k(x) \to f(x)$ we say

$f_k \to f$ ptwise.

Folland has a nice ex.

where

$f_k \to f$ ptwise but

$f_k' \not\to f'$

& $h_k \to h$ ptwise

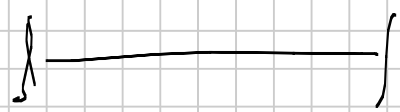
but

$\int_0^1 h_k \not\to \int_0^1 h$

Another kind of example —

$$S_k = \{\underline{x - \lfloor x \rfloor} : x \in [\log k, \log(k+1)]\}$$

$$f_k(x) = \begin{cases} 1, & x \in S_k \\ 0, & x \notin S_k. \end{cases}$$



what is the probability

$$f_k(x) = 1 \,? = \log(k+1) - \log(k)$$

$$f_k(x) = 0 \,? \quad 1 - \left(\log(k+1) - \log(k)\right)$$

$$= \log\left(1 + \frac{1}{k}\right) \longrightarrow 0$$

$$= 1 - \log\left(1 + \frac{1}{k}\right).$$

$$\longrightarrow 1$$

I would $f_k \to 0$
in probability

On other hand
for <u>no</u> x does
$f_k(x) \to 0$

x

as every i's in inf many S_k.

Uniform Convergence

HW 318

#1, 3, 5