Jan 30

Note Title 1/30/2009

Oriented Curves

C - oriented smooth curve.

$\vec{F}$ - vector field

$F : \mathbb{R}^k \to \mathbb{R}^k$

defined $\int_C \vec{F} \cdot d\vec{x}$

$g : [a, b] \to \mathbb{R}^k$ —
parametrization

$$\int_a^b \vec{F}(g(t)) \cdot \vec{g}'(t)\, dt$$

Green's Thm.

$\mathbb{R}^2$ —

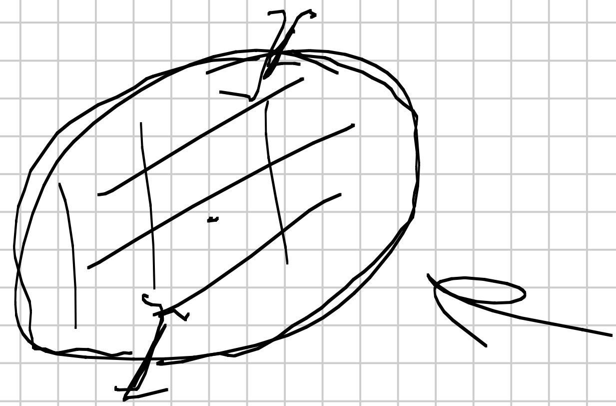
$S \subseteq \mathbb{R}^2$ -

S is "regular"
if S - compact
+ $S = \overline{(S^\circ)}$
S - closure of its interior

$\overline{\text{S has "piecewise}}$
smooth" boundary
if $\partial S = \bigcup_{i=1}^{m} C_i$
curves C_i - each +

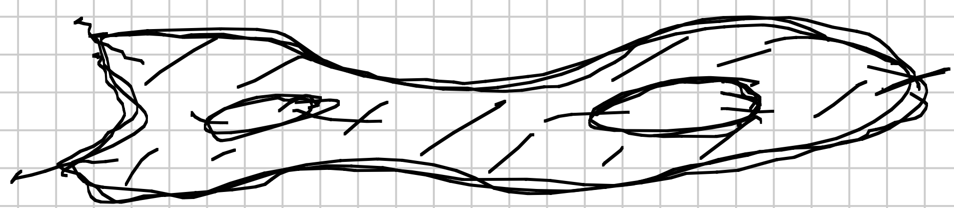
C^1 - param. curve.

Ex

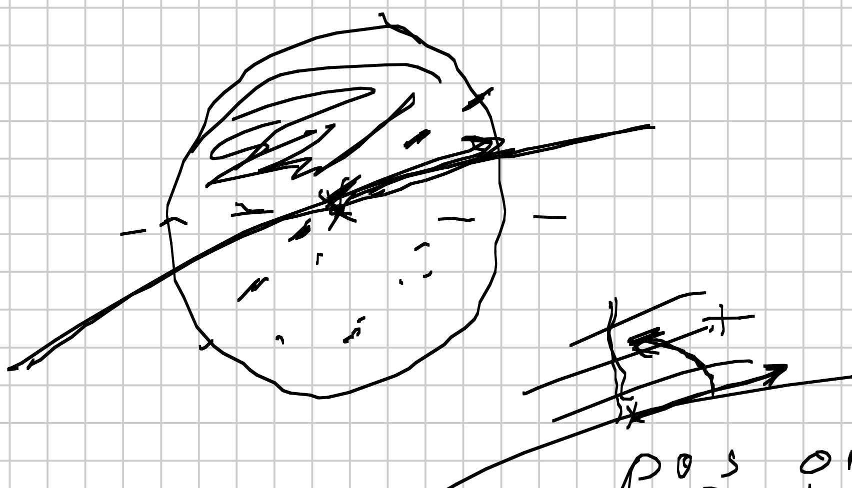


+ $C_i \cap C_j$ $i \neq j$

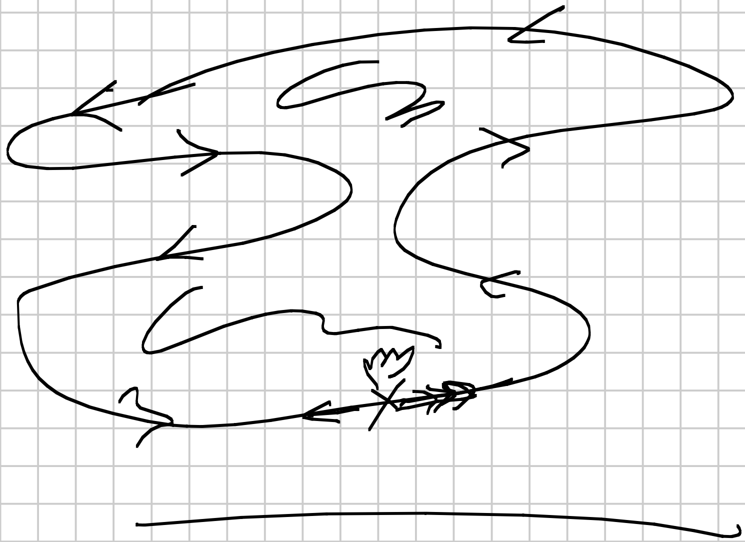
= at most endpts.



The boundary of such a region can be oriented.

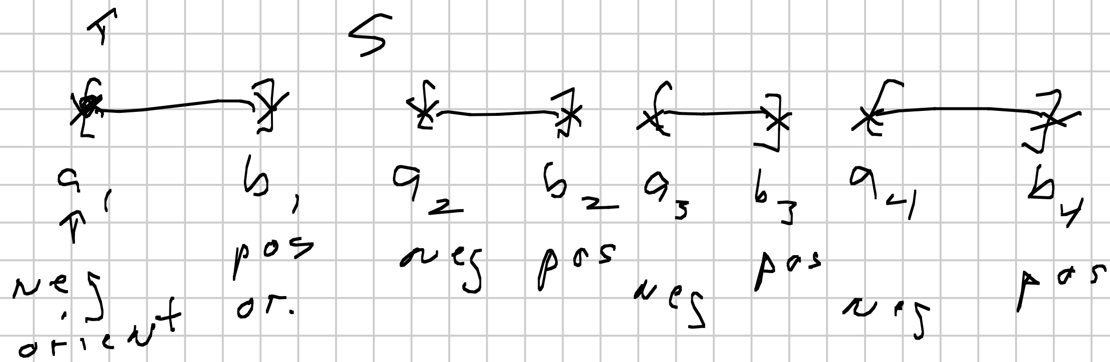


pos orientation of $\partial(S)$



$\mathbb{R}$

Regular set.



$$\int_S f'(x)\,dx$$

$$\longrightarrow f(b_1) - f(a_1) + f(b_2) - f(a_2) + f(b_3) - f(a_3) + f(b_4) - f(a_4).$$

Thm

Green's Thm.

Suppose $S \subseteq \mathbb{R}^2$ is regular & has piecewise smooth boundary & $\vec{F}$ is a C^1-vector field on S.

Then

$$\int_{\partial S} F \cdot dx = \iint_S \underbrace{\left(\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2} \right)} dx\,dy$$

$$\begin{vmatrix} \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} \\ \partial F_1 & \partial F_2 \end{vmatrix}$$

$$\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2}$$

1 dim

$$\int_{\partial S} f \, \delta x = \int_S \frac{df}{dx} \, dx$$

$$\left| \frac{\partial f}{\partial x} \right|$$

$$\int_{\partial S} F \cdot dx = \iint_S \left(\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2} \right) dA$$

pos. orient.

1) Suppose G.T. is true for $\vec{F}$, $\vec{G}$ on a domain S.

Then true for $\vec{F} + \vec{G}$

$$\int_{\partial S} (F+G)\cdot dx = \int_{\partial S} F \cdot dx + \int_{\partial S} G \cdot dx$$

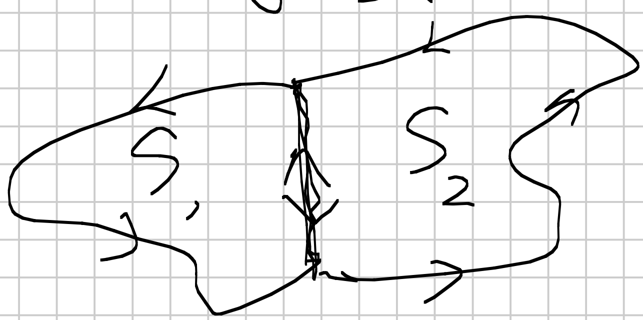
$$\iint_S \left(\frac{\partial (F_2+G_2)}{\partial x_1} - \frac{\partial (F_1+G_1)}{\partial x_2} \right) dA$$

$$= \iint_S \left(\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2}\right) dA + \iint_S \left(\frac{\partial G_2}{\partial x_1} - \frac{\partial G_1}{\partial x_2}\right) dA$$

$$S = S_1 \cup S_2$$

$S_1 + S_2$ only int. on

$\partial' S$



if G.T. True on S_1 & S_2
then true on $S_1 \cup S_2$.

Means we can
<u>reduce</u> the
<u>problem</u>

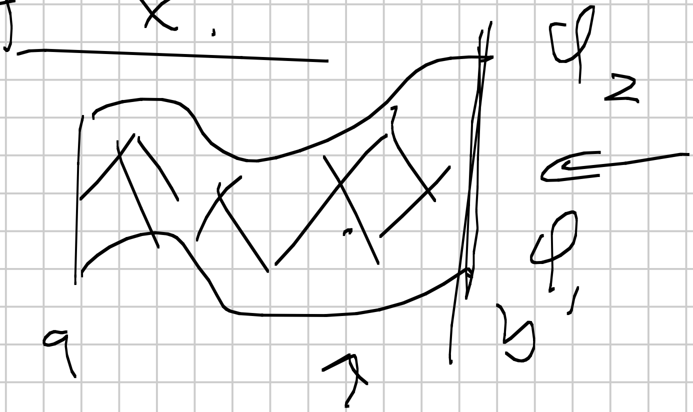
$$\vec{F}(x,y) = (F_1(x,y), F_2(x,y))$$

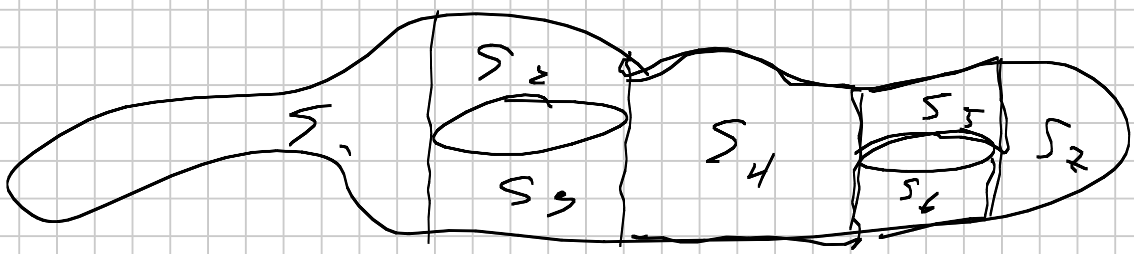
$$\vec{P}(x,y) = (F_1(x,y), 0)$$

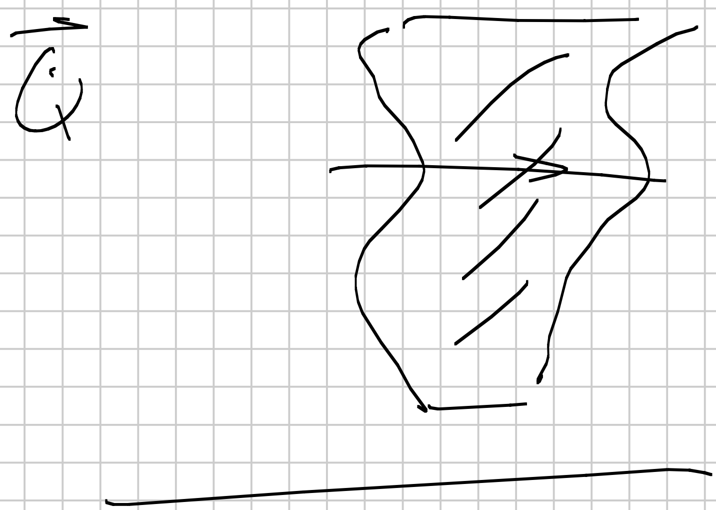
$$\vec{Q}(x,y) = (0, F_2(x,y))$$

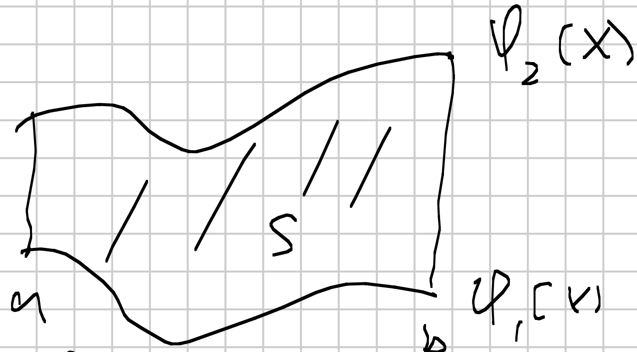
$$\vec{P}(x, y) = (F_1(x, y), 0)$$

Suppose S is a region between graphs of 2 functions of x.




S1
S2
S4
S6
S7



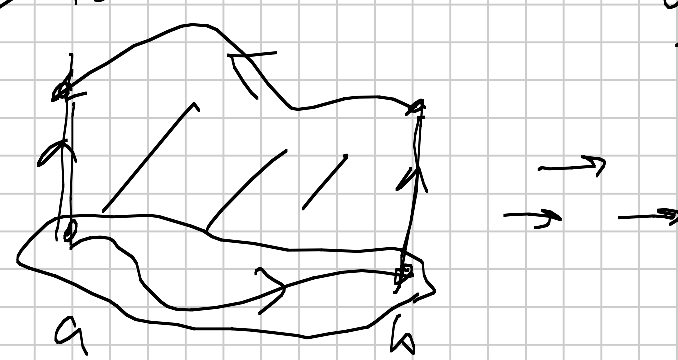


$$\int_a^b \int_{\varphi_1(x)}^{\varphi_2(x)} \left(- \frac{\partial F_1}{\partial x_2} \, dx_2 \right) dx_1 \qquad \left(\begin{array}{l} x_1 = x \\ x_2 = y \end{array} \right)$$

$$\iint_S \left(\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2} \right) dA$$

$$\int_a^b \left(F_1(\varphi_2(x)) - F_1(\varphi_1(x)) \right) dx$$

$$= \underbrace{\int_a^b F_1(\varphi_1(x))\,dx}_{\text{lower}} - \underbrace{\int_a^b F_1(\varphi_2(x))\,dx}_{\text{upper}}$$



Lower ∂.

$(x, \varphi_1(x))$

$$\int_{\text{lower } \partial} \vec{F} \cdot dx = \int_a^b F_1(\varphi_1(x))\, dx$$

Have shown G.T. holds for vector fields with just a 1st term.

Same true for $\vec{Q}$ – with only a 2nd coord,

$\Rightarrow$ Full G.T.