Jan 21

Integration

heiroglyphics -

$$\int_a^b f(x)\,dx$$

2 inputs

information

$$\int_0^1 x^2\,dx, \quad \int_{\cancel{[0,1]}}$$

$(a+b)$ & f

Domain of ~~int~~ function —

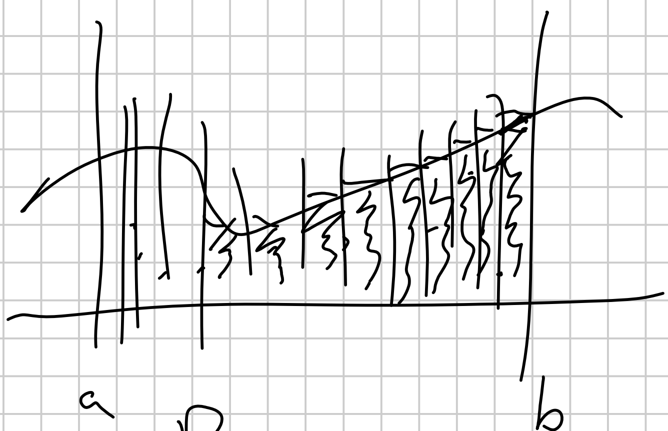
Lebesgue Integration

Domain of int —
in Riemann Integral —
Interval

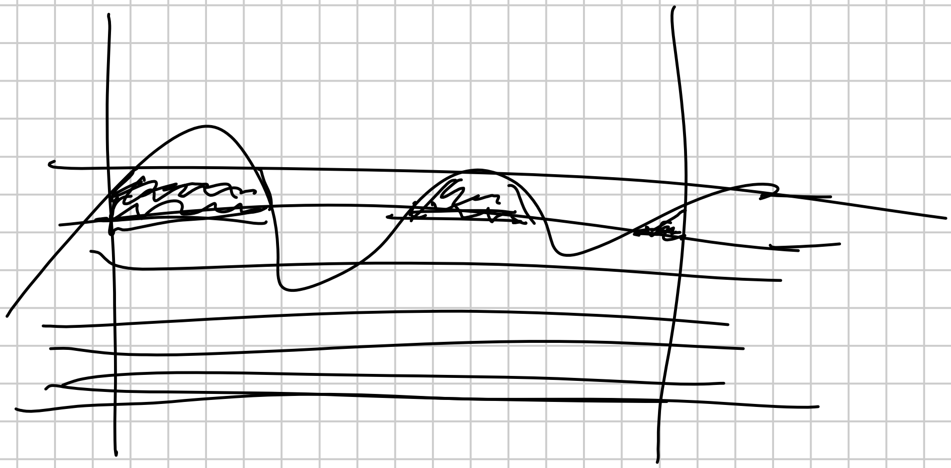
function — "almost" continuous

$$\int_{[a,b]} f(x)\,dx$$

~~Lebesgue~~



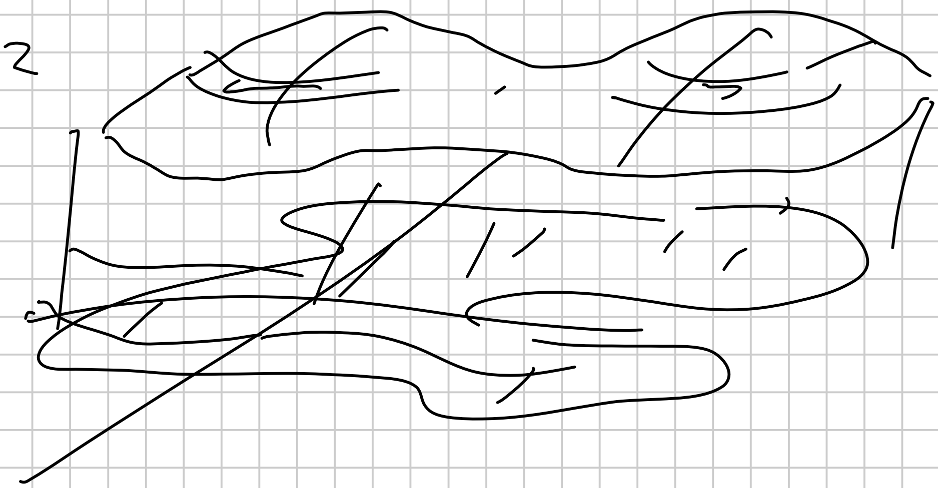
Riemann



Lebesgue –

I need a way to "measure" how big a set is.

$\mathbb{R}^2$



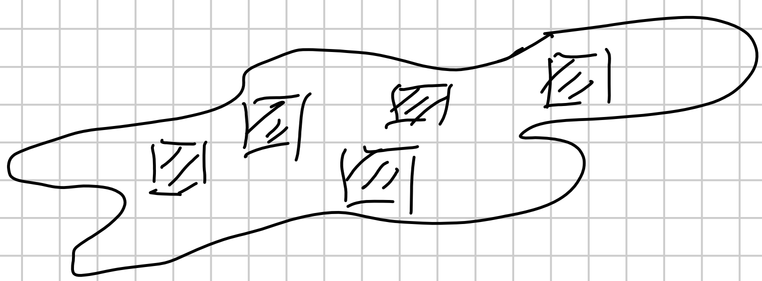
How can I <u>measure</u> the size of a set?

<u>in $\mathbb{R}^2$?</u>

1) Rectangles – base × ht. Mult.

2) Finite disj. union of rect.

Add



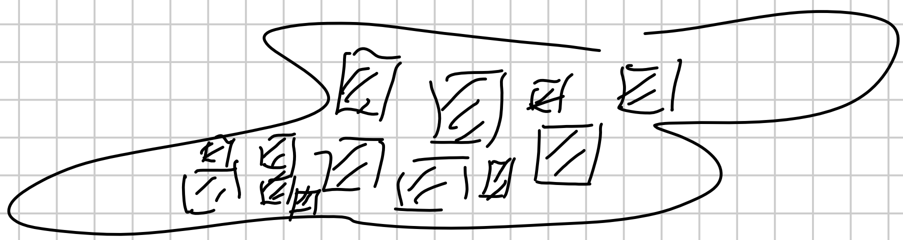
If $\mathcal{O}$ is open then

$\mathrm{Area}(\mathcal{O})$

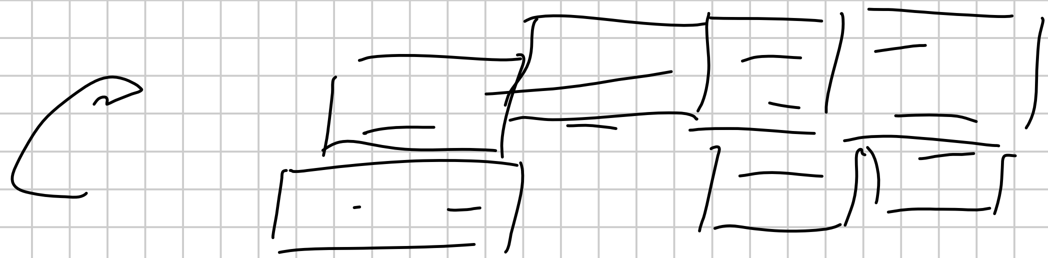
$= \sup\left(\sum_{i=1}^{n} \mathrm{area}(R_i)\right.$ where

$R_1 \ldots R_n$ are disj.

rect. in $\left.\mathcal{O}\right)$.



Compact:



Cover with disj. rect

$$\text{Area}(C) = \inf\Big(\sum_{i=1}^{n} \text{Area } R_i,\ \underline{R_i - \text{disj.}} + \text{cover } C\Big)$$

<u>General set</u>:

<u>Outer</u> Area (measure) m^*

<u>Inner</u> Area (measure) m_*

$$m^*(S) = \inf(\text{Area}(O), \ O \text{ open} \ \& \ S \subseteq O)$$

$$m_*(S) = \sup(\text{Area}(C), \ C \text{ - compact} \ \& \ C \subseteq S)$$

If $m^*(S) = m_*(S) \neq \infty$.

We say S is Lebesgue measurable.

If $m^*(S \cap [-n,n])$

$= m_*(S \cap [-n,n])$

Then S is Lebesgue measurable

1) open sets & compact are measurable –

2) The measurable sets form a "σ – algebra"

Closed under compliments & countable unions & countable intersections

if $A = \bigcup_{i=1}^{\infty} A_i$ & A_i are disj. & measurable

then

$$m(A) = \sum_{i=1}^{\infty} m(A_i).$$