Feb 27

Note Title 2/27/2009

Exam - next fri.

1) 3 problems

& 1 will be a HW problem -

Green's Thm
Div Thm
Stokes Thm

$\Big\}\ \mathbb{R}^2$ & $\mathbb{R}^3$.

Absolute Conv.

Conditional Conv.

$$\sum_{n=0}^{\infty} a_n x^n$$

Cases of
Cond. Conv.

1) Alternating series:

Suppose a_n are real #'s

1) $|a_{n+1}| \leq |a_n|$

2) $\lim_{n \to \infty} a_n = 0.$

3) $a_n a_{n+1} < 0$

i.e. the a_n's alternate sign.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

Then $\sum_{n=1}^{\infty} a_n$ converges.

Bigger result:

Summation by Parts –

Lemma $\{a_n\}$ & $\{b_n\}$ are real sequences – $(n = 0, 1, 2 \cdots)$

Let $a_n' = a_n - a_{n-1}$, $n = 1 \cdots$

$$\mathcal{B}_n = b_0 + b_1 + \cdots + b_n$$

then

$$\sum_{n=0}^{k} a_n b_n = a_k \mathcal{B}_k - \sum_{n=1}^{k} a_n' \mathcal{B}_{n-1}$$

pf

$$b_0 = \mathcal{B}_0$$

$$b_n = -\mathcal{B}_{n-1} + \mathcal{B}_n$$

$$\sum_{n=0}^{k} a_n b_n = a_0 b_0 + a_1 b_1 + \cdots + a_k b_k$$

$$= a_0 \mathcal{B}_0 - a_1 \mathcal{B}_0 + a_1 \mathcal{B}_1 - a_2 \mathcal{B}_1 + a_2 \mathcal{B}_2$$

$$+ \cdots - a_k B_{k-1} + a_k B_k$$

$$= (a_0 - a_1) B_0 + (a_1 - a_2) B_1 + \cdots + (a_{k-1} - a_k) B_{k-1} + a_k B_k$$

$$= a_k B_k - \left\{ (a_1 - a_0) B_0 + (a_2 - a_1) B_1 + \cdots + (a_k - a_{k-1}) B_{k-1} \right\}$$

$$= a_k B_k - \sum_{n=1}^{k} a_n' B_{n-1}$$

Thm (Dirichlet)

Suppose a_n & b_n are numer. sequences &

1) a_n are decreasing to 0.

$(a_{n+1} \le a_n$ & $\lim a_n = 0)$

2) $B_n = b_0 + b_1 + \cdots + b_n$ are bounded in absolute value ind. of n. Then

$$\sum_{n=0}^{\infty} a_n b_n \text{ converges.}$$

Ex $\sum \frac{(-1)^n}{n}$.

$a_n = \frac{1}{n}$ dec to 0.

$b_n = (-1)^n$

$B_n = \underbrace{-1+1-1+1 \cdots \pm 1}$.

bounded in abs value by 1.

$\Rightarrow$ CONV.

Ex $a_n \downarrow 0$, $a_{n+1} \leq a_n$

+ $\lim a_n = 0$

then $\sum_{n=1}^{\infty} (-1)^n a_n$

conv.

$1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} \cdots$

$a_n = \frac{1}{n}$

$b_n = 1, 1, -1, -1, 1, 1, -1, -1 \cdots$

pf of Dirichlet

$$\sum_{n=0}^{k} a_n b_n$$

$$= \underbrace{a_k B_k}_{\uparrow} - \underbrace{\sum_{n=1}^{k} a_n' B_{n-1}}_{\uparrow}$$

So if we can show both rt hand terms converge we're done.

$$|a_k B_k| = |a_k| \, |B_k|$$

Let C be a bound for $|B_k|$

$$+0 \le |a_k B_k| \le |a_k| C$$

$$= a_k C \longrightarrow 0$$

$$\Rightarrow \lim_{k \to \infty} a_k B_k = 0.$$

$$\sum_{n=1}^{k} a_n' B_{n-1}$$

① $a_n \downarrow 0$, $a_n' = (a_n - a_{n-1})$

<u>so</u> $a_n' < 0$.

Show $\sum_{n=1}^{\infty} a_n' B_{n-1}$ conv. absolutely.

$$\sum_{n=1}^{k} |a_n' B_{n-1}|$$

$$= \sum_{n=1}^{k} (a_{n-1} - a_n) |B_{n-1}|$$

$$\le \sum_{n=1}^{k} (a_{n-1} - a_n) C$$

$$= C \sum_{n=1}^{k} (a_{n-1} - a_n)$$

$$= C\left((a_0 - \cancel{a_1}) + (\cancel{a_1} - \cancel{a_2}) + (\cancel{a_2} - \cancel{a_3}) \right.$$

$$\left. \cdots + (\cancel{a_{k-1}} - a_k)\right)$$

$$= C(a_0 - a_k).$$

<u>So</u> $\displaystyle\sum_{n=1}^{\infty} |a_n' B_{n-1}|$

$$\leq \lim_{k\to\infty} C((a_0 - \cancel{a_k})^{0})$$

$$= C a_0$$

So $\displaystyle\sum_{n=1}^{\infty} a_n' B_{n-1}$

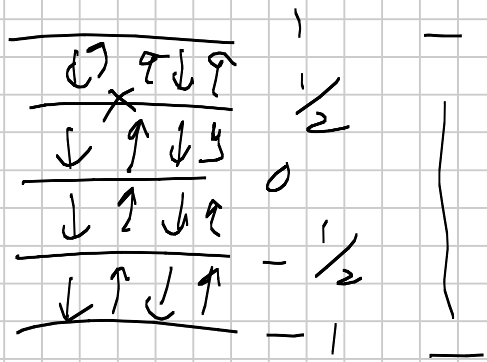
is abs conv.

+ proof dont

$$\sum a_n b_n$$

$$a_n = \frac{1}{n}$$

$$b_n = \frac{1}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{1}{2}, 0$$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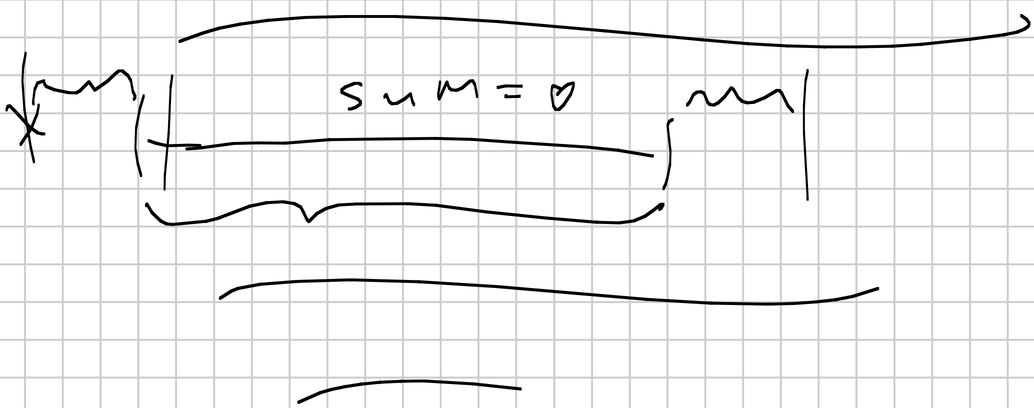
sum = 0