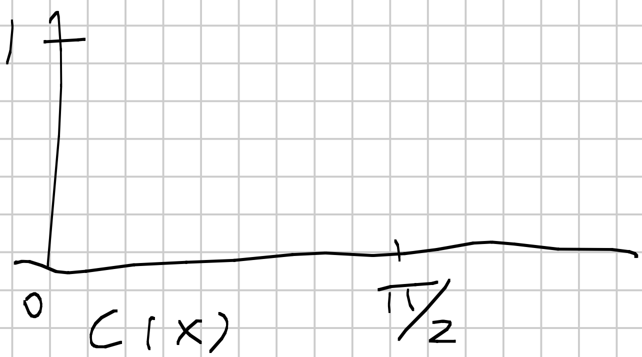
Note Title 4/8/2009

Apr 8



1) $C(0) = 1$

2) for $x \in [0, \pi/2)$, $C(x) > 0$

3) $S'(x) = C(x) \Rightarrow S$ is strictly increasing on $[0, \pi/2)$

4) $S(0) = 0 \Rightarrow S(x) > 0$ <u>on</u> $(0, \pi/2)$

5) $s(\pi/2) \geq 0 \Rightarrow s(\pi/2) = 1$

6) $c'(x) = -s(x)$ so

c is decreasing

on $(0, \pi/2)$.

7) $c'(0) = -s(0) = 0$

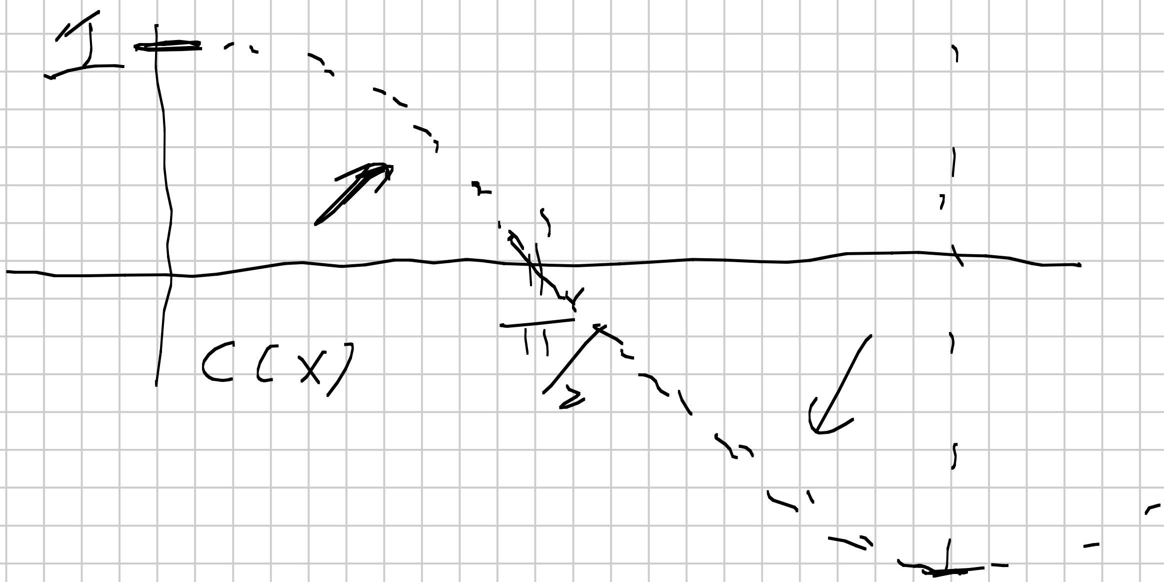
$c'(\pi/2) = -s(\pi/2) = -1$

8) $c''(x) = -c(x)$ – neg

on $(0, \pi/2)$

$\Rightarrow$ c is conc. down

on $(0, \pi/2)$



Fourier Series

Series in sin(mx)

& cos(mx)

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

We can alternatively write this as

$$\sum_{n=-\infty}^{\infty} c_n e^{inx}, \quad x \in \mathbb{R}.$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{inx} = \cos nx + i \sin nx$$

$$\sum_{n=-\infty}^{\infty} C_n(\cos nx + i\sin nx)$$

$$= C_0 + \sum_{n=1}^{\infty}\Big((C_n + C_{-n})\cos(nx) + (C_n - C_{-n})\sin(nx)\Big)$$

$$\left.\begin{aligned} a_n &= C_n + C_{-n} \\ \underline{b_n} &= C_n - C_{-n} \end{aligned}\right]$$

$$a_0 = C_0 + C_{-0} = \underline{\underline{2C_0}}$$

$$C_0 = \frac{a_0}{2}$$

$$c_n = \frac{a_n}{2} - i\frac{b_n}{2}$$

$$c_{-n} = \frac{a_n}{2} + i\frac{b_n}{2} \qquad n \geq 1$$

Observation:

$$e^{inx} = (e^{ix})^n$$

$$e^{ix} = z, \quad |z| = 1$$

pt on unit circle in $\mathbb{C}$.

$$\sum_{n=-\infty}^{\infty} c_n e^{inx} =$$

$$\sum_{n=-\infty}^{\infty} c_n z^n$$

On the unit circle in $\mathbb{C}$,

$$z^{-1} = \bar{z}$$

$$\sum_{n=-\infty}^{\infty} c_n z^n = \sum_{n=0}^{\infty} c_n z^n +$$

$$\sum_{n=1}^{\infty} c_{-n} \bar{z}^n .$$

Periodic functions

$f : \mathbb{R} \to \mathbb{R}$ -

period $p > 0$.

with $f(x+p) = f(x)$

Change of variable

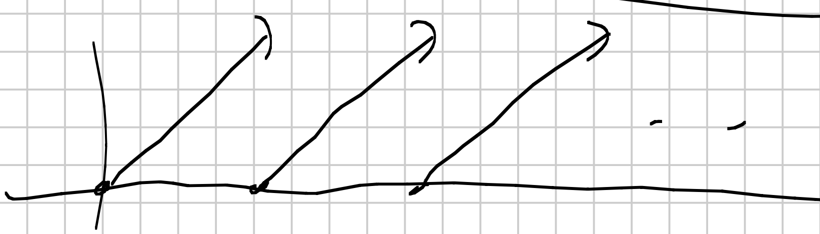
$$y = \frac{2\pi x}{p}$$

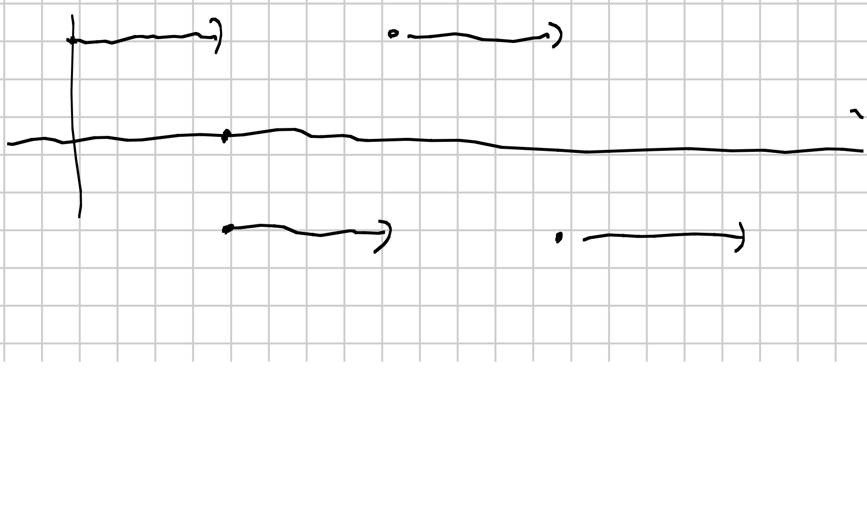
$$g(y) = f\left(\frac{py}{2\pi}\right) = f(x)$$

$$g(y+2\pi) = f\left(\frac{p(y+2\pi)}{2\pi}\right)$$

$$= f\left(\frac{py}{2\pi} + p\right)$$

$$= f\left(\frac{py}{2\pi}\right) = g(y).$$





Assume: f is
2π-periodic & piecewise
cont.

Assume f has
rt & left limits
everywhere & on
all but finitely many
pts in $[0, 2\pi)$, they
agree w/ $f(x)$.

1) Given f how
do we find the

coeff?

2) Does the Fourier series converge?

1) Suppose $\vec{v}_1, \vec{v}_2$ is an orthonormal basis for $\mathbb{R}^2$ & I write

$$\vec{u} = a\,\vec{v}_1 + b\,\vec{v}_2$$

how do I find a & b?

$\vec{u} \cdot \vec{v}_1 = a \quad , \vec{u} \cdot \vec{v}_2 = b$

vectors $\longrightarrow$ functions

I need an inner prod.
& in $\mathbb{C}$.

$$\langle f, g \rangle = \int_0^{2\pi} f(x)\,\bar{g}(x)\,dx$$

both 2π-per.
& $\mathbb{C}$-valued,

$$\langle f, f \rangle = \int_0^{2\pi} f(x)\,\bar{f}(x)\,dx$$

$$= \int_0^{2\pi} |f(x)|^2 dx = \|f\|_2$$

$$\langle e^{inx}, e^{imx} \rangle$$

$$= \int_0^{2\pi} e^{inx} e^{-imx} dx$$

$$= \int_0^{2\pi} e^{i(n-m)x} dx$$

$$= \int_0^{2\pi} \cos(n-m)x \, dx + i \int_0^{2\pi} \sin(n-m)x \, dx$$

Case 1, $n = m$, $= \dfrac{2\pi}{\pi}$

Case 2, $n \neq m$

$$\frac{\sin(n-m)x}{n-m}\Bigg|_0^{2\pi} - i\,\frac{\cos(n-m)x}{n-m}\Bigg|_0^{2\pi}$$

$$= 0$$

We have (almost) an orthonormal set of fctns.

This <u>means</u>

$$f = \sum_{n=-K}^{K} c_n e^{in\theta}$$

$$\Rightarrow \quad c_m = \frac{1}{2\pi} \langle f, e^{im\theta} \rangle$$

$$\langle f, e^{im\theta} \rangle$$

$$= \sum_{n=-K}^{K} \int_0^{2\pi} \underline{c_n e^{in\theta} \, e^{-im\theta}}$$

$$= 2\pi c_m$$

So The only possible choice for $c_m = \langle f, e^{imx} \rangle / 2\pi$