M417, Fall 2009, Second hourly exam

This is a closed book, closed notes exam. Show all your work. For full credit you must show complete arguments.

Prob. 1 (30 pts)
Define f in $\mathbb{R}^2$ by
$$f(x, y) = 2x^3 - 3x^2 + 2y^3 + 3y^2.$$
Find all critical points of f and determine whether they are local minima, maxima or saddle points.

Prob. 2 (35 pts)
A surface Σ in $\mathbb{R}^3$ is given parametrically as the image of the map
$$F(s, t) = (s + t, t^2, t^3).$$

a) Show that F is one-to-one.

b) At what points $(a, b, c) = F(s_0, t_0)$ of the surface can you guarantee that in a neighborhood of (a, b, c), Σ is a smooth surface?

c) Find a nonzero vector $\vec{n}$ perpendicular to the surface at such a point (a, b, c).

Prob. 3 (35 pts)
Use the Taylor series for $\sin(x)$ and $\cos(x)$ to find the value c for which
$$\lim_{x \to 0} \frac{\sin(x) - x\cos(x) - cx^3}{x^5}$$
exists. For this value of c evaluate the limit. Be sure to justify your work. No credit will be give for using L'hôpital's rule.