M417, Fall 2009, First hourly exam

This is a closed book, closed notes exam. Show all your work. For full credit you must show complete arguments.

Prob. 1 (30 pts)

Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ and we find that the directional derivative in the direction $\vec{u} = (u_1, u_2)$ at the point $(1, 2)$ is given by

$$\partial_{\vec{u}} f(1, 2) = u_1 + u_2^2.$$

a) What is $\nabla f(1, 2)$?

b) Is f differentiable at $(1, 2)$? Why or why not?

Prob. 2 (35 pts)

A sequence is defined recursively by $x_1 = 1$ and $x_{k+1} = \sqrt{1 + x_k}$.

a) Show that the x_k are monotone increasing and bounded above by 3 and hence converge.

b) Find the limit of the x_k.

c) If x_1 is changed to 3 does the sequence still converge? Why or why not?

Prob. 3 (35 pts)

Suppose $f : (0, 1] \to \mathbb{R}$ is continuous and the $\lim_{x \to 0^+} f(x) = L$ exists. Prove that f is uniformly continuous.