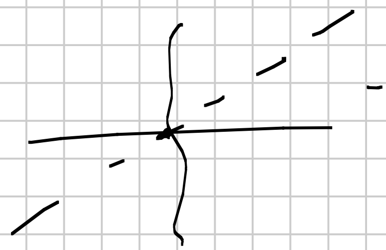
Sept 9

Note Title 9/9/2009

$$\frac{x^2 y}{x^4+y^2}$$



$$y = cx$$

$$f_c(x) = f(x, cx)$$

$$= \frac{cx^3}{x^4+c^2x^2} = \frac{cx}{x^2+c^2}.$$

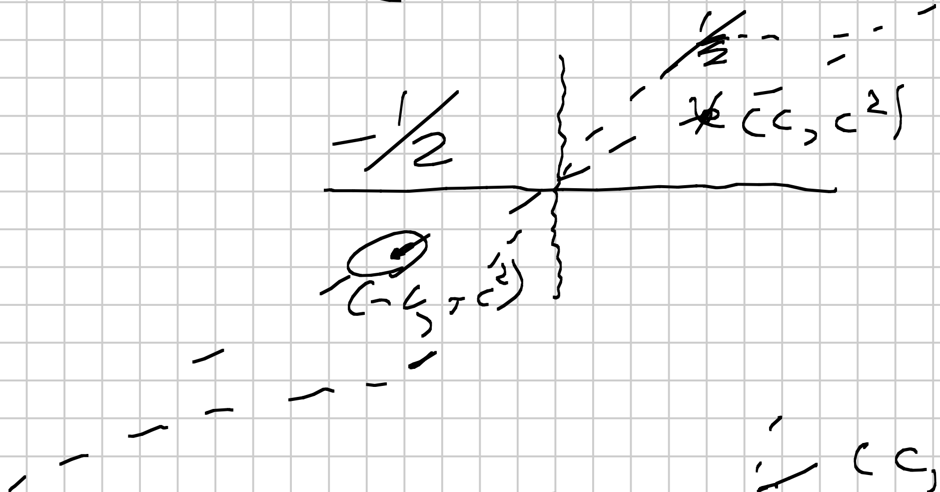
where are max & min?

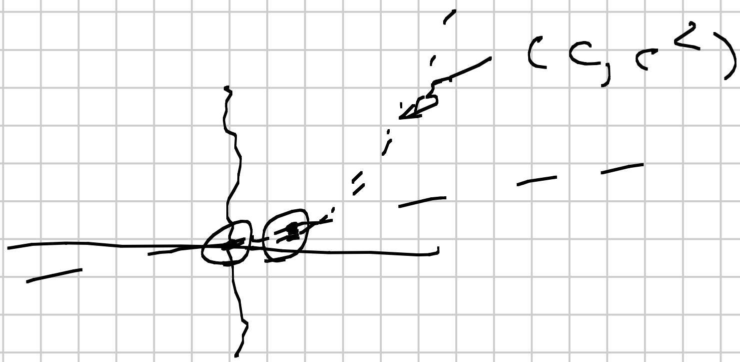
$$f_c'(x) = \frac{(x^2+c^2)c - cx(2x)}{(x^2+c^2)^2}$$

$$= \frac{c^3 - cx^2}{(x^2+c^2)^2} = 0$$

$$c^3 = cx^2$$
$$x^2 = c^2$$
$$x = \pm c.$$
$$f_c(c) = \frac{1}{2}, \quad f_c(-c) = -\frac{1}{2}$$

$c \neq 0$





$\exists$ pts, (c, c^2), converging to $(0,0)$ where $f(c, c^2) = \frac{1}{2}$ & $(-c, -c^2)$ where $f(-c, -c^2) = -\frac{1}{2}$.

(a) $\dfrac{x^2+y}{\sqrt{x^2+y^2}}$, 0 at $(0,0)$.

Show the limit at
(0,0) does not exist.

for $y=0$, $x=0$

$$f(x\ )=\frac{x^2}{\sqrt{x^2}}=|x|$$

$$f(\ y)=\frac{y}{\sqrt{y^2}}=\begin{cases}1, & y>0\\ -1, & y<0\end{cases}$$

$$\frac{y}{|y|}$$

f etc.

2 b) $f(x,y)=\dfrac{3x^5-xy^4}{x^4+y^4}$

Show $\lim\limits_{\vec{x}\to\vec{0}} f(\vec{x})=0$.

$x^4=x^4$

$x^4\le x^4+y^4$

$\& \ |x^5| \leq |x|\,|x^4+y^4|$

So $\underbrace{\dfrac{|x^5|}{|x^4+y^4|}} \leq |x|$

$y^4 = y^4$

$y^4 \leq x^4 + y^4$

$|xy^4| \leq |x|\,|x^4+y^4|$

$$\frac{|xy^4|}{|x^4+y^4|} \leq |x|$$

$$\left| \frac{3x^5 - xy^4}{x^4+y^4} \right|$$

$$\leq \frac{3|x^5|}{|x^4+y^4|} + \frac{|xy^4|}{|x^4+y^4|}$$

$$\leq \underline{\underline{4}}\,|x|.$$

Let $\varepsilon > 0$. Set $\delta = \dfrac{\varepsilon}{4}$

If $\|(x,y)\| < \delta$

then $|x| < \delta$.

i.e $|x| < \frac{\varepsilon}{4}$

So $|f(x,y)| = \left| \frac{3x^5 - xy^4}{x^2+y^2} \right| \le 4|x| < \varepsilon$

$f : A \to B$

if f is one to one & onto then f has an inverse.

$f(C) = \{f(x) : x \in C\}$.

Always have an inverse on sets.

$f^{-1}(D) = \{x \mid f(x) \in D\}$.

A B

"Pullback" of D.

1) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

2) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$

3) $f^{-1}(A^c) = (f^{-1}(A))^c$.

Thm $f : D \to \mathbb{R}^m$

$D \subseteq \mathbb{R}^n$.

f is continuous on D

iff for every open set $U \subseteq \mathbb{R}^m$,

$f^{-1}(U)$ is open.