Sept 28

Note Title 9/28/2009

HW p52 #2
67 #1,4
69 #3

Diff.

$$f(x) = \boxed{\text{const}} + \boxed{\text{linear Term}} + \boxed{\text{error}}$$

Approx near $\vec{a}$

$$f(x) = f(\vec{a}) + \boxed{} + E(x).$$

We want

$$\lim_{\vec{x} \to \vec{a}} \frac{|E(\vec{x})|}{\|\vec{x} - \vec{a}\|} = 0$$

$f : \mathbb{R}^n \to \mathbb{R}$

linear term

$$\sum_{i=1}^{n} c_i (x_i - a_i)$$

$$= \vec{c} \cdot (\vec{x} - \vec{a}).$$

$$(c_1, c_2 \cdots c_n)\begin{pmatrix} x_1 - a_1 \\ \vdots \\ x_n - a_n \end{pmatrix}$$

$$c_i = \frac{\partial f}{\partial x_i}(\vec{a})$$

$$f(\vec{x}) = f(\vec{a}) + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\vec{a})(x_i - a_i)$$

$$+ E(\vec{x})$$

$$\left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

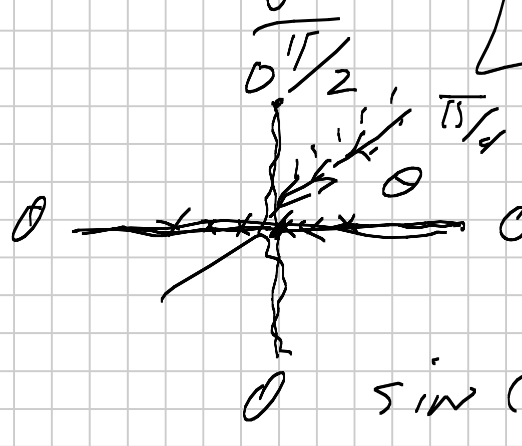
$$= \nabla f$$

$$= \operatorname{grad} f$$

$$f(x) = f(\vec{a}) + \nabla f \cdot (\vec{x} - \vec{a})$$

$$+ E(x)$$

Ex $f(x,y) = \begin{cases} 0 & \text{at } \vec{0} \\ \sin(2\theta), & \text{otherw} \end{cases}$



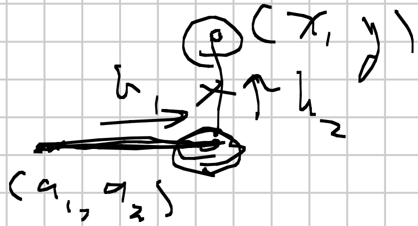
$$\sin(2\theta) = 2\sin\theta\cos\theta = \frac{2xy}{x^2+y^2}$$ //

Thm Suppose $f : D \to \mathbb{R}$, $D \subseteq \mathbb{R}^n$ & $\vec{a} \in D^{int}$. Assume all partial deriv. of f exist & are cont. on D. Then f is diff at $\vec{a}$.

pf. 2 dim

$$f : \mathbb{R}^2 \to \mathbb{R},$$

$$\left.\begin{aligned} \vec{a} &= (a_1, a_2) \\ (x, y) &= (a_1 + h_1,\ a_2 + h_2) \end{aligned}\right\} \in D$$



$$E(x) = \underbrace{f(\vec{x}) - f(\vec{a})} - \frac{\partial f}{\partial x}(\vec{a}) h_1 - \frac{\partial f}{\partial y}(\vec{a}) h_2$$

$$f(a_1 + h_1, a_2 + h_2) - f(a_1 + h_1, a_2)$$
$$+ f(a_1 + h_1, a_2) - f(a_1, a_2)$$

$$\frac{\partial f}{\partial y}(a_1 + h_1, a_2 + c_2) \cdot h_2$$
$(c_2$ - bet 0 & $h_2)$

$+$

$$\frac{\partial f}{\partial x}(a_1 + c_1, a_2) \cdot h_1$$
$(c_1$ - bet. 0 & $h_1)$

MVT in 1 var.

$$\left(\frac{\partial f}{\partial x}(a_1+c_1, a_2) - \frac{\partial f}{\partial x}(\vec{a})\right) h_1 + \left(\frac{\partial f}{\partial y}(a_1+h_1, a_2+c_2) - \frac{\partial f}{\partial y}(a)\right) h_2$$

$$= V \cdot (h_1, h_2)$$

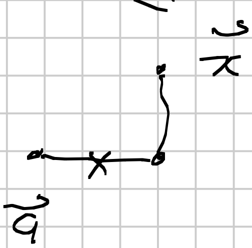
$$= E(x)$$

$$|E(x)| \le \|V\| \, \|(\vec{x}-\vec{a})\|$$

$$\frac{|E(x)|}{\|(\vec{x}-\vec{a})\|} \le \|V\|$$

1st comp of V

$$\lim_{\vec{x} \to \vec{a}} \left| \frac{\partial f}{\partial x}(a_1+c_1, a_2) - \frac{\partial f}{\partial x}(\vec{a}) \right|$$



as $\vec{x} \to \vec{a}$, $c_1 \to 0$

& $\frac{\partial f}{\partial x}$ is cont $\Rightarrow$

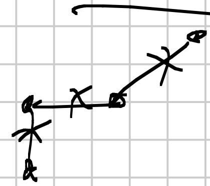
this lim. is $\underline{0}$.

2nd.

$$\left|\frac{\partial f}{\partial y}(a_1+h_1, a_2+c_2) - \frac{\partial f}{\partial y}(\vec{a})\right|$$

& again as $\vec{x} \to \vec{a}$

this $\to 0$



Directional Derivatives

$\vec{u}$ - unit vector

$\|\vec{u}\| = 1$

$$f_{\vec{u}}(t) = f(\vec{a} + t\vec{u})$$

$$f'_{\vec{u}}(0) = \partial_{\vec{u}} f(\vec{a})$$

if it exists

Suppose f is diff. at $\vec{a}$

$$\frac{f(\vec{a}+t\vec{u}) - f(\vec{a})}{t}$$

$$= \frac{\nabla f(\vec{a}) \cdot t\vec{u} + E(a+t\vec{u})}{t}$$

$$= \nabla f(\vec{a}) \cdot u - \underbrace{\frac{E(a+t\vec{u})}{t}}$$

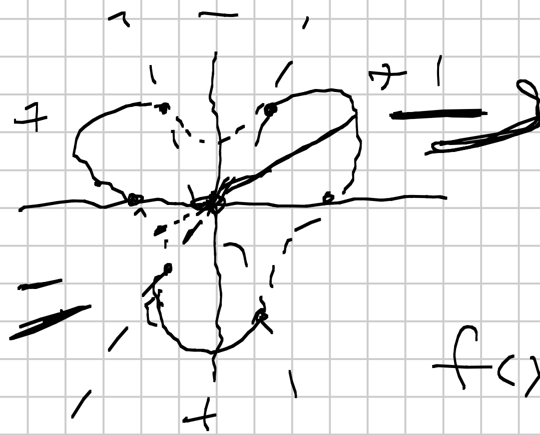
$$\left| \frac{E(\vec{a}+t\vec{u})}{t} \right|$$

$$= \frac{|E(\vec{a}+t\vec{u})|}{|(\vec{a}+t\vec{u})-\vec{a}|}$$

0

So $\partial_{\vec{u}} f(\vec{a}) = \nabla f(\vec{a}) \cdot \vec{u}$.

Ex $f(x,y) = r\sin 3\theta$



$$f(x,y) = \frac{3x^2y - y^3}{x^2+y^2}$$

$$(0,-1) = \nabla f(\vec{0})$$

Enough