Oct 5

Note Title 10/5/2009

Office hrs
Today 1-2
Tues 11-12
Wed 11-12

3:30 - W117
- Ext Review -

Gradient

$f: D \to \mathbb{R}$, $D \subseteq \mathbb{R}^n$

f is diff-

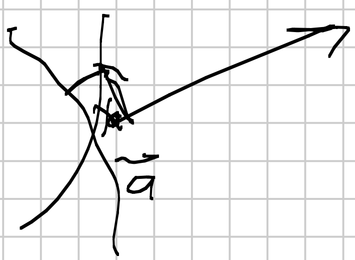
∇f

① at $\vec{a}$, $\nabla f(\vec{a})$ is dir. where the dir deriv. is max (if $\neq 0$)

② $\nabla f(\vec{a})$ is $\perp$ the tangent plane to the level sfc.

$$f(\vec{x}) = f(\vec{a}) \leftarrow$$

$(\nabla f(\vec{a}) \neq 0)$



$\nabla f(\vec{a})$

c) MVT

If $\vec{a}, \vec{b}$ & the line segment $\vec{a}(1-t) + \vec{b}t,\ t \in [0,1]$ are in D then $\exists\ \vec{c}$

$$c = \vec{a}(1-t_0) + \vec{b}t_0,\ t_0 \in (0,1)$$

$$\& \quad f(\vec{b}) - f(\vec{a}) = \nabla f(\vec{c}) \cdot (\vec{b} - \vec{a})$$

Cor If D is convex

$\&\ \|\nabla f(\vec{x})\| \le M \quad \forall x \in D$

then $\forall\ \vec{a}, \vec{b} \in D$,

$$|f(\vec{b}) - f(\vec{a})| \leq M \|\vec{b} - \vec{a}\|.$$

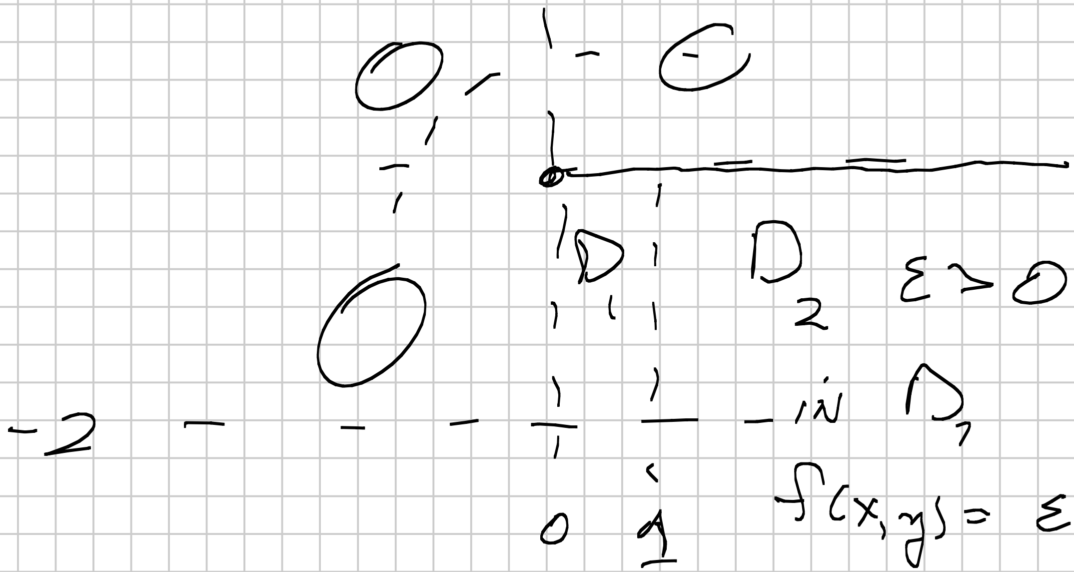
$\forall\, \vec{a}, \vec{b}\ \exists\, \vec{c}$ with

$$f(\vec{b}) - f(\vec{a}) = \nabla f(\vec{c}) \cdot (\vec{b} - \vec{a})$$

$$|f(\vec{b}) - f(\vec{a})| \leq M \|\vec{b} - \vec{a}\|$$

Ex - Convexity matters

$$D = \mathbb{R}^2 - \{(x, 0) \mid x \geq 0\}$$



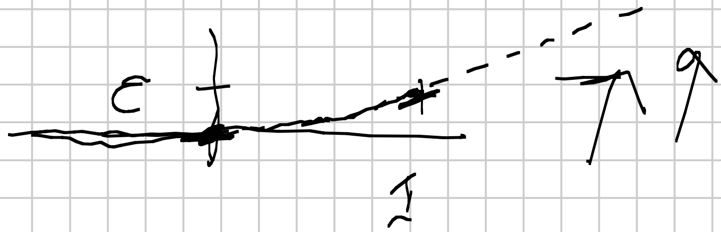
$\varepsilon > 0$

in D_1

$$f(x,y) = \varepsilon x^2$$

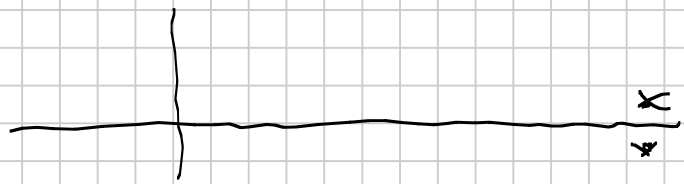
in D_2

$$f(x,y) = \varepsilon + 2\varepsilon(x-1)$$



Class C^1

$$\|\nabla f\| \le 2\varepsilon$$



$$x = \frac{10^6}{\varepsilon} + 1$$

$$y_{1,2} = \begin{cases} + \text{smidge} \\ - \text{smidge} \end{cases}$$

$$|f(x, y_2) - f(x, y_1)|$$

$$= \varepsilon + 2\varepsilon(x-1)$$

$$> 10^6$$

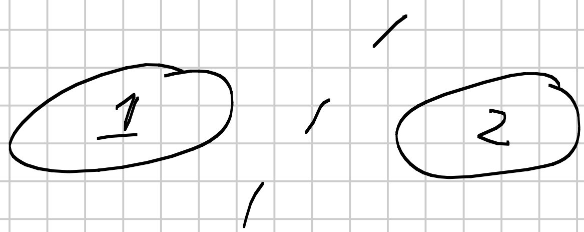
Cor - If D is convex & $\nabla f \equiv \vec{0}$ on D

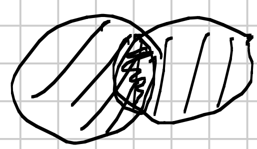
Then f is constant

on D.

$$|f(\vec{b}) - f(\vec{a})| \le M \underbrace{\|\vec{b} - \vec{a}\|}_{=0}$$

$$\Rightarrow f(\vec{b}) = f(\vec{a})$$





Thm If D is open & connected & $\nabla f \equiv 0$ on D

Then f is a constant on D.

pf Equivalence relations.

$\vec{a} \, C \, \vec{b}$ if $\vec{a}, \vec{b} \in D$

& $\exists$ a list of open balls.

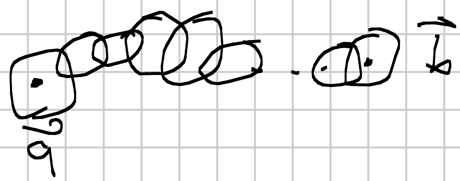
$$B_{r_1}(a_1), \; B_{r_2}(a_2) \dots B_{r_n}(a_n)$$

$$r_i > 0$$

$$a_1 = a, \quad a_n = b$$

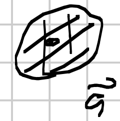
$$B_{r_i}(a_i) \subseteq D$$

$$\& \quad B_{r_i}(a_i) \cap B_{r_{i+1}}(a_{i+1}) \neq \emptyset$$



① $\vec{a} \, C \, \vec{a}$

reflexive



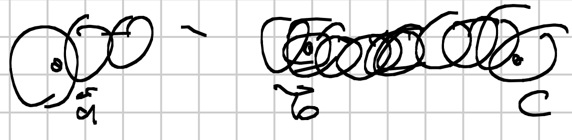
② if $\vec{a} \, C \, \vec{b}$ then $\vec{b} \, C \, \vec{a}$

symmetric

③ if $\vec{a} \, C \, \vec{b}$ & $\vec{b} \, C \, \vec{c}$

then $\vec{a} \, C \, \vec{c}$.

Transitive



C is an equiv. rel.

$\langle \vec{a} \rangle = \{ \vec{b} : \vec{a} \, C \, \vec{b} \}$

Classes are either = or disj.

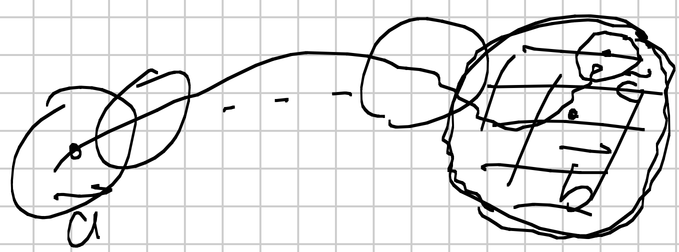
If $\vec{a} \, C \, \vec{b}$ then

$f(\vec{a}) = f(\vec{b})$

f is constant on $\langle \vec{a} \rangle$.

Show only one class.

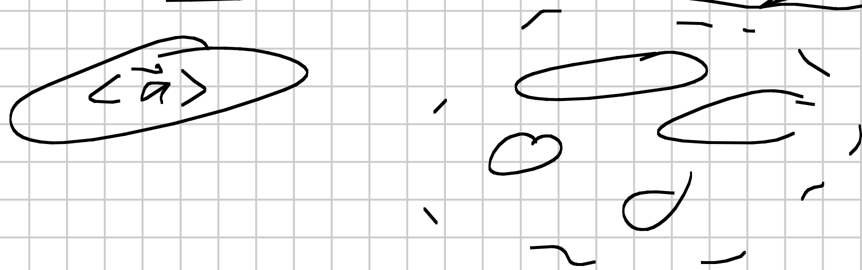
Show $\langle \vec{a} \rangle$ is open.



every pt in the ball around $\vec{b}$ is in $\langle \vec{a} \rangle$.

$\Rightarrow \langle \vec{a} \rangle$ - open

Suppose there are more than one equiv. class



$\langle \vec{a} \rangle = S_1$ - open

Union of the rest $= S_2$ - open

1) $D = S_1 \cup S_2$

2) S_1 & $S_2 \neq \emptyset$

3) $\overline{S_1} \cap S_2$

$S_1 \subseteq S_2^c$ - closed

so $\overline{S_1} \subseteq S_2^c$

$\Rightarrow \overline{S_1} \cap S_2 = \emptyset$

& $S_1 \cap \overline{S_2} = \emptyset$

That disc. D

$\Rightarrow \Leftarrow$

So only one class
& f is const.
on D