Oct 28

Note Title 10/28/2009

$$f(x,y) = (x-1)(x^2-y^2)$$

$$\partial_x f = 2x(x-1) + (x^2-y^2)$$
$$= 3x^2 - y^2 - 2x$$

$$\partial_y f = -2y(x-1).$$

$y = 0$

$3x^2 - 2x = 0$

$x(3x-2) = 0$

$x = 0$ or, $x = 2/3$

$(0,0)$, $(2/3, 0)$

$x = 1$

$-y^2 + 1 = 0$

$y = \pm 1$

$(1,1)$, $(1,-1)$.

$$\partial_{xx} f = 6x - 2$$
$$\partial_{xy} f = -2y$$
$$\partial_{yy} f = -2(x-1)$$

pt	H	
$(0,0)$	$\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$	Saddle

$(2/3, 0)$ $\begin{bmatrix} 2 & 0 \\ 0 & 2/3 \end{bmatrix}$ loc min

$(1, 1)$ $\begin{bmatrix} 4 & -2 \\ -2 & 0 \end{bmatrix}$ saddle

det $= -4$

$(1, -1)$ $\begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$ saddle

det $= -4$

Last Section

$$\mathbb{R}^n \xrightarrow[f]{} \mathbb{R}^m \xrightarrow[g]{} \mathbb{R}^p$$

$$D(g \circ f)(\vec{a})$$

$$= D(g)(f(\vec{a})) \cdot D(f)(\vec{a})$$

Matrix prod

Next Chapt.

Implicit + Inverse

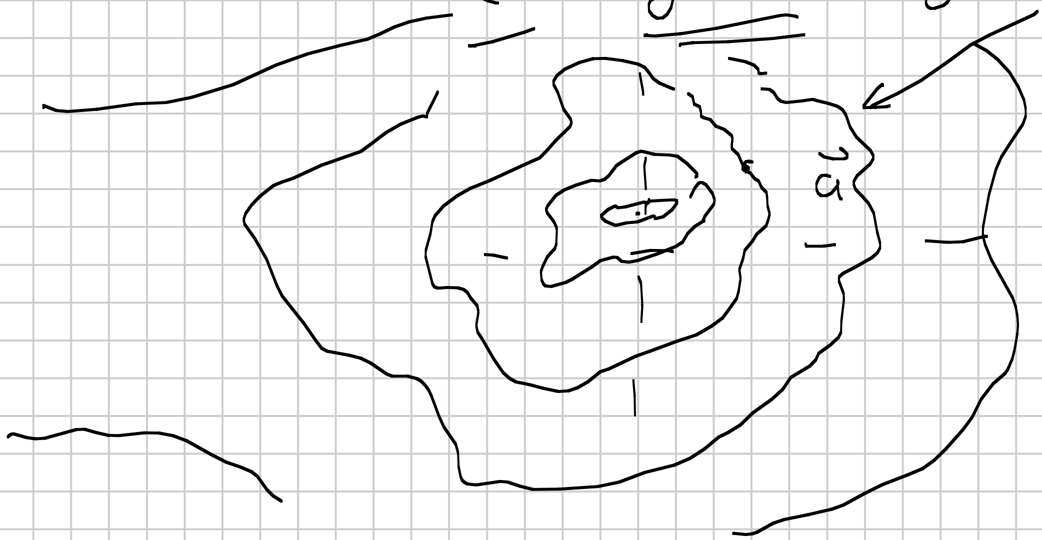
Function Thms

Implicit fct. Thm.

$g(\vec{x})$, $g : \mathbb{R}^n \to \mathbb{R}$.

$\vec{a}$

$\{\vec{x} : g(\vec{x}) = g(\vec{a})\}$



This should be an $(n-1)$ dim. set

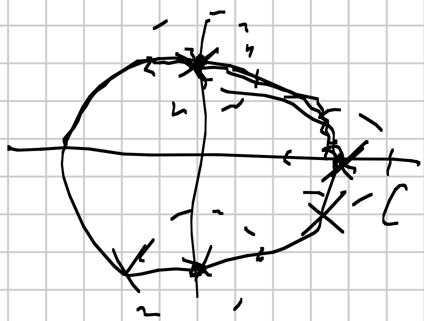
perhaps can solve for one var. in terms of others.

$$g(x,y) = x^2 + y^2$$

$$\vec{a} = (0,1)$$

$$g(\vec{a}) = 1$$

$$x^2 + y^2 = 1$$



$$y = \sqrt{1-x^2}$$

$$g(x,y) = g(\vec{a})$$

$$F(\vec{x}) = g(\vec{x}) - g(\vec{a}) = 0$$

$$F(x_1, \dots, x_n).$$

Assume $F(\vec{a}) = 0$

Look for a function

$$f(x_1 \dots x_{n-1})$$

so that in nbhd of $(a_1, \dots a_{n-1})$

$$F(x_1 \dots x_{n-1}, f(x_1 \dots x_{n-1})) = 0.$$

Thm (Implicit function thm in 1 var)

$\mathbb{R}^{n+1}$, $(\underline{x_1, \dots, x_n}, y)$

$F : S \to \mathbb{R}$, $S \subseteq \mathbb{R}^{n+1}$, open

$F(\vec{a}, b) = 0$

$\partial_y F(\vec{a}, b) \neq 0$.

Then $\exists$ numbers $r_0, r_1 > 0$ so that

a) For each $\vec{x} \in B_{r_0}(\vec{a})$ $\exists$ a unique y in $|y - b| < r_1$ with $F(\vec{x}, y) = 0$.

Let $y = f(\vec{x})$

b) f is C^1 on $B_{r_1}(\vec{a})$

& $$\partial_{x_j} f(\vec{x}) = -\frac{\partial_{x_j} F(\vec{x}, f(\vec{x}))}{\partial_y F(\vec{x}, f(\vec{x}))}.$$

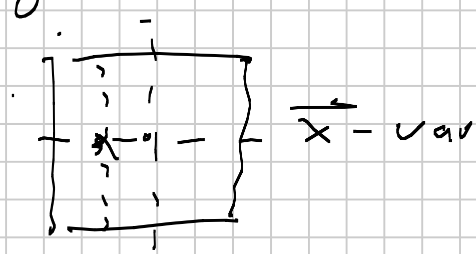
pf Assume $\partial_y F(\vec{a}, b) > 0$

(rep. F by $-F$ if not).

$\partial_y F$ is cont., $\exists\, r_1$

& if $\|\vec{x}-\vec{a}\| < r_1$ & $|y-b| < r_1$

$\Rightarrow \partial_y F(\vec{x}, y) > 0$.



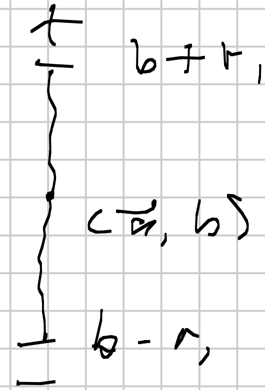
For each $\vec{x} \in B_{r_1}(\vec{a})$,

F is a st. inc. fctn of y.

$F(\vec{a}, b) = 0$ so

$F(\vec{a}, b + r_1) > 0$

$F(\vec{a}, b - r_1) < 0$



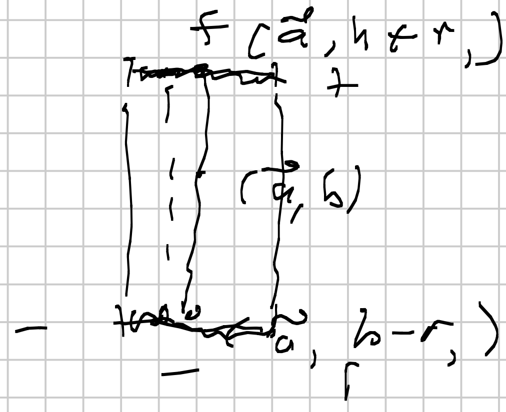
F is <u>cont.</u>

so $\exists\, r_0 > 0$, $r_0 < r_1$

$\vec{x} \in B_{r_0}(\vec{a})$

$F(\vec{x}, b + r_1) > 0$

$F(\vec{x}, b - r_1) < 0$

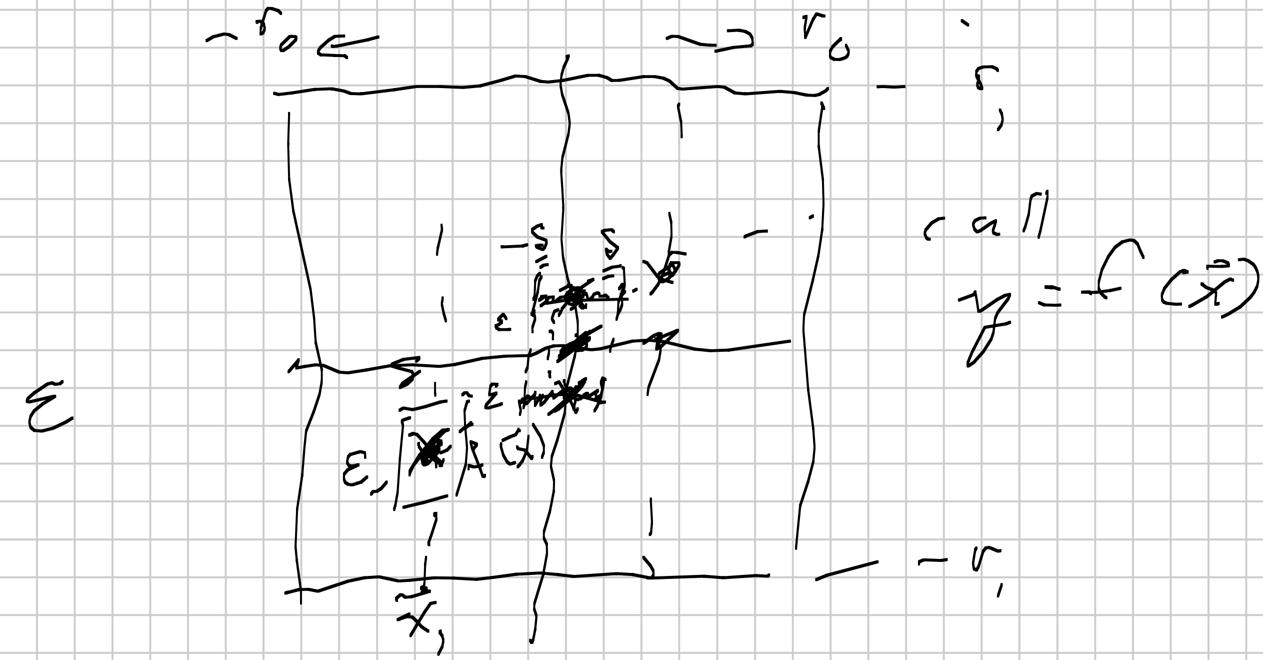


$\vec{x} \in B_{r_1}(\vec{a})$ - line

from $(\vec{x}, b - r_1)$, $(\vec{x}, b + r_1)$

F goes from − to +
& is st. inc.

Hence for each $\vec{x} \in B_{r_1}(\vec{a})$
there is a unique
$y \in [b-r_1, b+r_1]$,
$F(\vec{x}, y) = 0$



f is cont.