Oct 2

Note Title 10/2/2009

Ext. Review
3:30 W117

Geometric orientation

Curves -

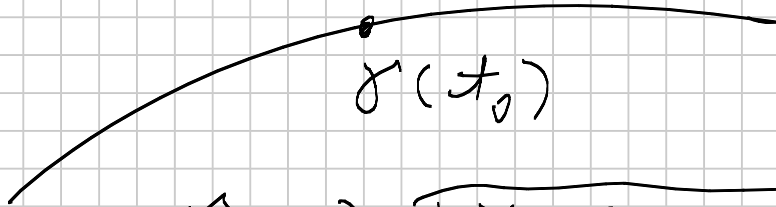
Parametrized curve -

$\gamma : [a,b] \to \mathbb{R}^n$

γ - continuous.

If γ is diff. at t_0 + $\gamma'(t_0) \neq \vec{0}$

Then $\gamma'(t_0)$ is a tangent vector to curve at $\gamma(t_0)$.

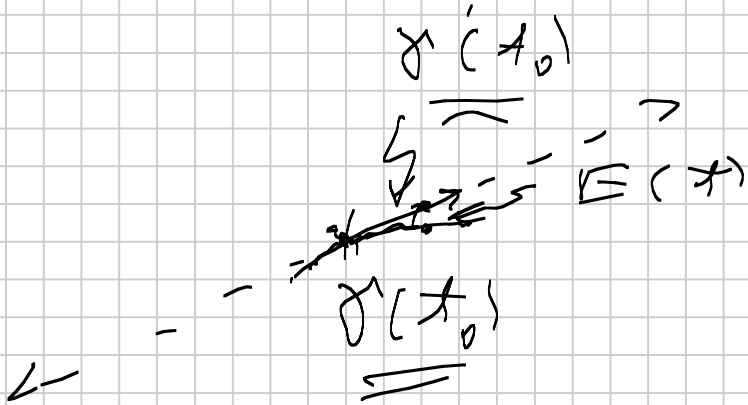


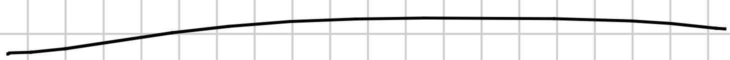
$$\gamma(t) = \gamma(t_0) + \gamma'(t_0)(t - t_0) + E(t)$$

$$\gamma(t) = (\gamma_1(t), \ldots, \gamma_n(t))$$

$$\gamma'(t) = (\gamma_1'(t), \ldots, \gamma_n'(t))$$

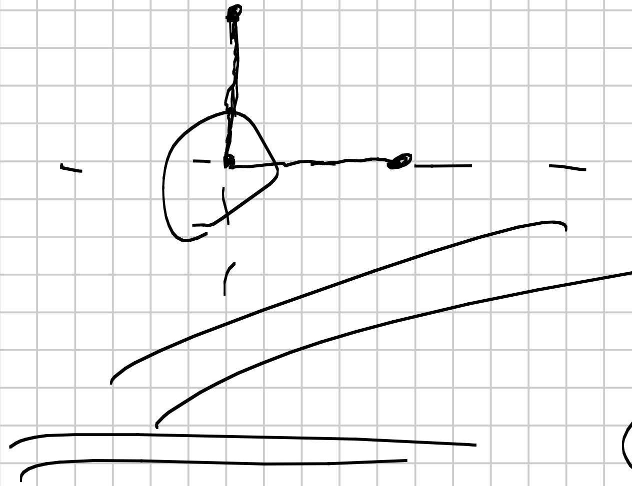
$$\frac{\|E(t)\|}{|t - t_0|} \to 0$$



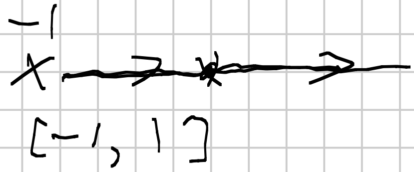


$$\gamma(t) = \begin{cases} (0, t^2), & t \in [-1, 0] \\ (t^2, 0), & t \in [0, 1] \end{cases}$$

$$\gamma'(t) = \begin{cases} (0, 2t), & t \in [-1, 0] \\ (2t, 0), & t \in [0, 1] \end{cases}$$



$(t^3, 0)$

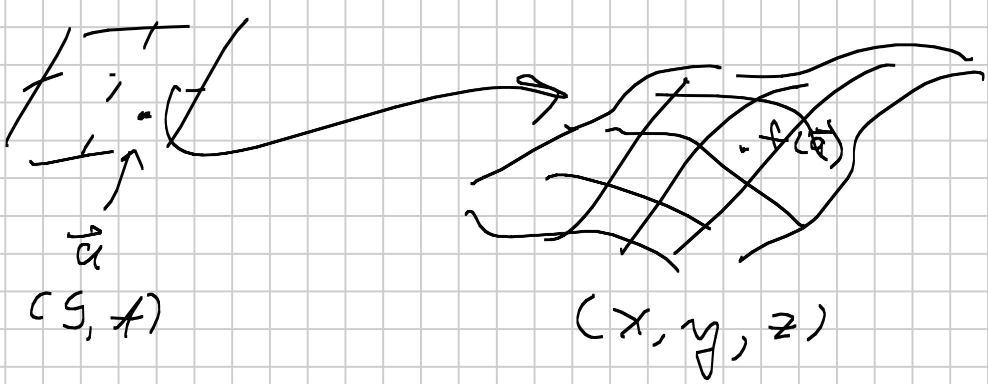


Tan planes in $\mathbb{R}^3$

Surface

$f : \mathbb{R}^2 \to \mathbb{R}^3$ - Cont.

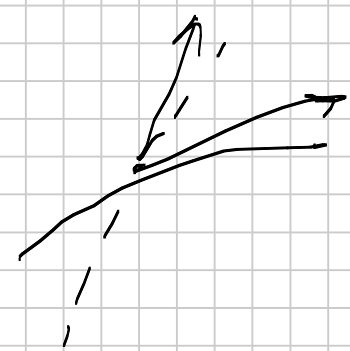
- Range is the sfc



$$f(s,t) = f(\vec{a}) + Df(\vec{a})\cdot((s,t) - \vec{a}) + E(s,t)$$

$$Df = \begin{bmatrix} \frac{\partial f_1}{\partial s} & \frac{\partial f_1}{\partial t} \\ \frac{\partial f_2}{\partial s} & \frac{\partial f_2}{\partial t} \\ \frac{\partial f_3}{\partial s} & \frac{\partial f_3}{\partial t} \end{bmatrix}$$

If the 2 columns are lin ind then they span



the tangent plane

Level sfc.

$f : \mathbb{R}^3 \to \mathbb{R}$.

Diff.

$$f(\vec{x}) = f(\vec{a}) + \nabla f(\vec{a})\cdot(\vec{x} - \vec{a}) + E(\vec{x})$$

Fact: $\nabla f(\vec{u})$, if it isn't 0, is the dir in which f increases most rapidly.

$$\partial_{\vec{u}} f(\vec{a}) = \nabla f \cdot \vec{u}$$

$$\leq \|\nabla f(\vec{a})\| \cancel{\|\vec{u}\|}$$

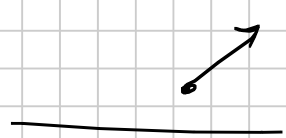
Can this $= \|\nabla f(\vec{a})\|$?

yes but only if

$$\vec{u} = c \, \nabla f(\vec{a})$$

$$\vec{u} = \frac{\nabla f(\vec{a})}{\|\nabla f(\vec{a})\|}$$

(smallest is at $-\dfrac{\nabla f(\vec{a})}{\|\nabla f(\vec{a})\|}$



Level Sfces

$$f : \mathbb{R}^n \to \mathbb{R}$$

$$\Sigma_c = \{\vec{x} : f(\vec{x}) = c\}.$$

in $\mathbb{R}^3$, should be a sfc.

What is the tan plane to Σ_c going to be?

suppose $\gamma : [a,b] \to \mathbb{R}$ is a curve on Σ_c.

$$f(\gamma(t)) = c.$$

Diff

$$\underbrace{\nabla f(\gamma(t)) \cdot \gamma'(t)}_{\text{Chain rule}} = 0$$

$\gamma'(t)$ is tangent to curve.

So tangent vectors to surface are $\perp$ to $\nabla f(\gamma(t))$.

MVT

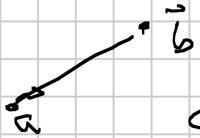
Suppose f is diff on $D \subseteq \mathbb{R}^n$, $\vec{a}, \vec{b} \in D$ & line segment connecting them is in D. Then $\exists\ \vec{c}$ on segment s.t.

$$f(\vec{b}) - f(\vec{a}) = \nabla f(\vec{c}) \cdot (\vec{b} - \vec{a})$$

Pf.

$$\gamma(t) = \vec{a}(1-t) + \vec{b}(t)$$

$$t \in [0,1]$$



$$g(t) = f(\gamma(t))$$

cont. on $[0,1]$

diff on $(0,1)$

so $g(1) - g(0) = g'(s)(1-0)$

$f(\vec{b}) - f(\vec{a}) = \nabla f(\gamma(s)) \cdot (\vec{b} - \vec{a})$

(C. R.)

Tomorrow is

My **60th**

Birthday!