Oct 12

Note Title 10/12/2009

Hi 60's - 70's A

45 - 60 B

40 - 45 C

1) $\vec{u} = (u_1, u_2)$

$\partial_{\vec{u}} f(1,2) = u_1 + u_2^2$

a) $\nabla f(1,2)$?

$\nabla f(1,2)$

$= (\partial_x f(1,2), \partial_y f(1,2))$

$\cancel{\partial_{\vec{u}} f(1,2) = \nabla f(1,2) \cdot u}$.

$$\partial_x f(1,2) = \partial_{(1,0)} f(1,2)$$
$$= 1 + 0^2 = 1$$
$$\partial_y f(1,2) = \partial_{(0,1)} f(1,2)$$
$$= 0 + 1^2 = 1$$
$$\underline{\underline{(1,1)}}$$

b) If f is diff at $(1,2)$ then

$$\partial_{\vec{u}} f(1,2) = \nabla f(1,2) \cdot \vec{u}$$
$$= (1,1) \cdot (u_1, u_2)$$
$$= u_1 + u_2$$
$$\neq u_1 + u_2^2 \implies \Leftarrow$$

so f is <u>not</u> <u>diff</u>.

2) $x_1 = 1$, $x_{k+1} = \sqrt{1+x_k}$

a) Mono. inc. - induction

$S = \{k : x_{k+1} \geq x_k\}$

$x_2 = \sqrt{2} > 1 = x_1$

$\Rightarrow \underline{1 \in S}$

If $k \in S$,

$x_{k+1} \geq x_k$

$1 + x_{k+1} \geq 1 + x_k$

$\sqrt{1+x_{k+1}} \geq \sqrt{1+x_k}$

$\quad\; \shortparallel \qquad\qquad\; \shortparallel$

$x_{k+2} \qquad x_{k+1}$

$\checkmark \quad k+1 \in S.$

$\Rightarrow \underline{S = \mathbb{N}.}$

b) Let $S = \{k : x_k \leq 3\}$.

$x_1 = 1 \leq 3$

$\underline{1 \in S}$

if $k \in S$,

$x_k \leq 3$

$x_k + 1 \leq 4$

$x_{k+1} = \sqrt{1 + x_k} \leq 2 < 3.$

$\Rightarrow k+1 \in S$

b)

$x_{k+1} = \sqrt{1 + x_k}$

$\lim_{k \to \infty} x_{k+1} = \lim_{k \to \infty} \sqrt{1 + x_k}$

$\quad\;\; \shortparallel \qquad\qquad\quad \shortparallel$

$L = \sqrt{1 + L}$

$L^2 = 1 + L$

$L^2 - L - 1 = 0$

$L = \frac{1 \pm \sqrt{5}}{2}$

Must be > 1

$L = \frac{1 + \sqrt{5}}{2}$

c) $x_1 = 3, \; x_2 = \sqrt{3+1} = 2.$

$x_3 = \sqrt{3} < 2$

Can I show

x_k's decrease?

$x_1 \geq x_2$ ✓

Suppose $x_k \geq x_{k+1}$

$1 + x_k \geq 1 + x_{k+1}$

$\sqrt{1+x_k} \geq \sqrt{1+x_{k+1}}$

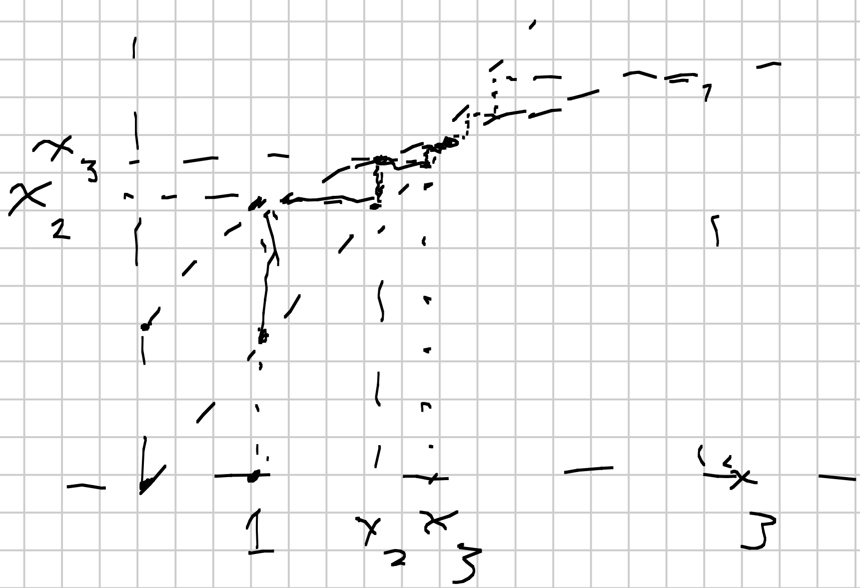
so $x_{k+1} \geq x_{k+2}$

Dec.

0 - lower bd.

So Conv.

$f(x) = \sqrt{1+x}$



3) $f : (0,1] \to \mathbb{R}$

1) Cont.

2) $\lim_{x \to 0^+} f(x) = L$

exists

Show f is unif. cont.

Let $\varepsilon > 0$.

Select $\delta_1 > 0$ so if

$0 < x \leq \delta_1 \Rightarrow |f(x) - L| < \varepsilon/2$.

On $[\delta_1, 1]$, f is unif cont. as interval is cpct.

Means $\exists\ \delta_2 > 0$,

$x, y \in [\delta_1, 1]$

$+ |x, y| < \delta_2$

then $|f(x) - f(y)| < \varepsilon/2$.

$\delta = \min(\delta_1, \delta_2)$.

Suppose $|x-y| < \delta$.

1) both x, y are $< \delta_1$

$\Rightarrow |f(x) - L| < \varepsilon/2$

& $|f(y) - L| < \varepsilon/2$

So $|f(x) - f(y)| < \varepsilon$.

2) both $x, y \geq \delta_1$

in $[\delta_1, 1]$

& $|f(x) - f(y)| < \varepsilon/2 < \varepsilon$

3) $\underbrace{x < \delta_1}$ $\leq y$

$|f(x) - f(\delta_1)| < \varepsilon/2$

& $|f(y) - f(\delta_1)| < \varepsilon/2$

$|f(x) - f(y)| < \varepsilon$.

Another pf

Extend f to 0
by setting $f(0) = L$.

$f : [0,1] \to \mathbb{R}$

& is cont.

$\therefore$ unif cont

$\Rightarrow f$ is unif cont