Nov 9

Note Title 11/9/2009

HW p. 140 #7
p. 157 #1

F is a C^1 fctn
$F(0,0)=0$
Find a cond. on F
so that $F(F(x,y),y)=0$
can be solved near
$(0,0)$ for y as a C^1
fctn of x.

$G(x,y) = 0$

Want $\partial_y G(0,0) \neq 0$.
the IFT says
sol. exists

$$G(x,y) = F(F(x,y),y)$$

$$= F(u(x,y),\ v(x,y))$$

$$u(x,y) = F(x,y)$$
$$v(x,y) = y$$

$$\partial_y G(x,y)$$

$$= \frac{\partial F}{\partial u}(u,v) \cdot \frac{\partial u}{\partial y}(x,y)$$

$$+$$

$$\frac{\partial F}{\partial v}(u,v) \cdot \frac{\partial v}{\partial y}(x,y)$$

$$= \partial_x F(F(x,y),y) \cdot \partial_y F(x,y)$$

$$+$$

$$\partial_y F(F(x,y),y) \cdot 1$$

$$\partial_y (G)(0,0) = \partial_x F(0,0)\, \partial_y F(0,0)$$

$$+$$

$$\partial_y F(0,0) \neq 0$$

$$\partial_y F(0,0)(\partial_x F(0,0)+1) \neq 0$$

So $\partial_y F(0,0) \neq 0$

$\partial_x F(0,0) \neq -1$

Integration

1-dim

$I = [a, b]$ -

$f : I \to \mathbb{R}$

f - bded.

Approx - area under f.

A Partition is a list of pts

$$a = x_0 < x_1 < x_2 \cdots x_n = b$$

$$P = \{x_0, \ldots, x_n\}$$

(More natural

$$P = \{[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]\}$$

$$= \{ I_1, I_2 \cdots I_n).$$

In each $I_i = [x_{i-1}, x_i]$

let

$$m_i = \inf(f(x) : x \in I_i)$$

$$M_i = \sup(f(x) : x \in I_i)$$

$$\overline{S}(f,P) = \sum_{i=1}^{n} M_i (x_i - x_{i-1})$$

$$= \sum_{i=1}^{n} M_i \, \ell(I_i)$$

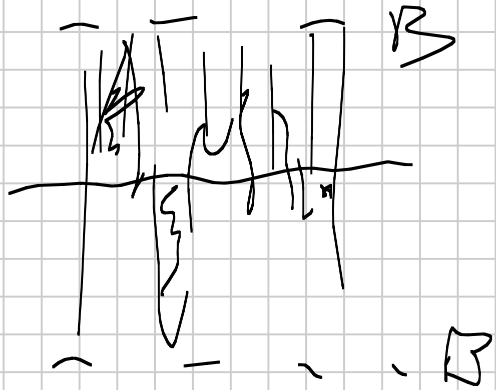
length

$$\underline{S}(f,P) = \sum_{i=1}^{n} m_i (x_i - x_{i-1})$$

$$= \sum_{i=1}^{n} m_i \, \ell(I_i).$$

If $|f(x)| \leq B$

$$-B(b-a) \leq \underline{S}(f,P) \leq \overline{S}(f,P) \leq B(b-a)$$



$$\overline{S}(f,P) = \sum_{i=1}^{n} M_i \, \ell(I_i)$$

$$\leq \sum_{i=1}^{n} B \, \ell(I_i)$$

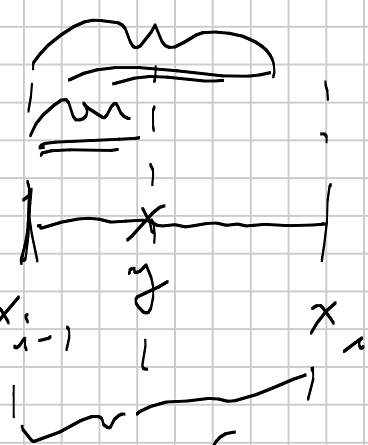
$= B(b-a)$.

Defn We say a partition Q "refines" P if $P \subseteq Q$.

Lemma If Q refines P then

$$\underline{S}(f,P) \le \underline{S}(f,Q) \le \overline{S}(f,Q) \le \overline{S}(f,P)$$

pf $\underline{S}(f,P) = \sum_{i=1}^{n} m_i \, \ell(I_i)$



$y \in Q$

(assume $Q = P \cup \{y\}$.

$\underline{S}(f,Q) - \underline{S}(f,P)$

$$= \inf\left(f(x) : x \in [x_{i-1}, y]\right)(y - x_{i-1})$$

$$+$$

$$\inf\left(f(x) : x \in [y, x_i]\right)(x_i - y)$$

$$- \inf\left(f(x) : x \in [x_{i-1}, x_i]\right)(x_i - x_{i-1})$$

$$\inf\left(f(x) : x \in [x_{i-1}, y]\right)$$

$$\geq \inf\left(f(x) : x \in [x_{i-1}, x_i]\right)$$

$$\geq 0$$

$$\overline{S}(f, P) = -\underline{S}(-f, P)$$

Cor: For any partitions P + Q

$$\underline{S}(f, P) \leq \overline{S}(f, Q).$$

pf Let $H = P \vee Q$

H refines P & Q

$$\underline{S}(f,P) \le \underline{S}(f,H) \le \overline{S}(f,H) \le \overline{S}(f,Q)$$

the values

$\{\underline{S}(f,P) : P \text{ a part}\}$

are bdd above.

Let $\underline{I}_a^b(f)$

be the sup of these values.

$\{\overline{S}(f,P) : P \text{ - a part}\}$

are bdd below

Let $\overline{I}_a^b(f)$ be the inf of these values

Defn We say f is Riemann integrable on $[a,b]$ if

$$\underline{I}_a^b(f) = \overline{I}_a^b(f)$$

& we call this the Riemann int of f

$$\int_a^b f(x)\,dx.$$