Nov 2

Note Title 11/2/2009

Grammar	20
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H/W p 119 #2, 5
p 124 1a, c, g

Investigate the pos. of solving

$F_1 = u^3 + xv - y = 0$

$F_2 \quad v^3 + yu - x = 0$

for 2 var. in terms of the other 2. near

$$\begin{array}{cccc} x & y & u & v \\ (0, & 1, & 1, & -1) \end{array}$$

is a sol.

$$DF = \begin{bmatrix} v & -1 & 3u^2 & x \\ -1 & u & y & 3v^2 \end{bmatrix}$$

at $(0,1,1,-1)$

$$\begin{bmatrix} -1 & -1 & 3 & 0 \\ -1 & 1 & 1 & 3 \end{bmatrix}$$

$$\begin{array}{cc} \bar{x} & \tilde{y} \\ \left[\begin{array}{c} A \end{array}\right. & \left|\ \boxed{B}\ \right] \end{array}$$

Any pair of columns are lin. ind. So <u>any</u> 2 var. can be solved for in a <u>nghd.</u>

$F : \mathbb{R}^4 \to \mathbb{R}^3$

$F(1,0,0,1) = 0$

$+ DF(1,0,0,1)$

$$= \begin{array}{c} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} \\ \begin{array}{cccc} x & y & u & v \end{array} \end{array}$$

Inv. when 3 of var can be solved for in terms of fourth.

w $\det\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \dfrac{0+2+0}{1+1+0} = 0$

?

y $\det\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} = \dfrac{-1}{1} = -2$

yes

z $\det\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \dfrac{-1}{1} = -2$

yes

x $\det\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \dfrac{-2}{1-1} = -2$

yes

Invert 2×2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} d/_{ad-bc} & -b/_{ad-bc} \\ -c/_{ad-bc} & a/_{ad-bc} \end{bmatrix}$$

Curves in $\mathbb{R}^2$:

1) Graph of $y = f(x)$
or $x = g(y)$

2) Sol. of $F(x,y) = 0$.

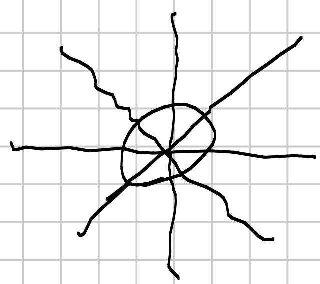
3) Parametrized as
some $\gamma(t)$

Smooth curves -
Want a tan vector
at every pt that
varies continuously.

in 1) if f or g is C^1
we have smoothness.

For 2) & 3) C'
is <u>not</u> enough.

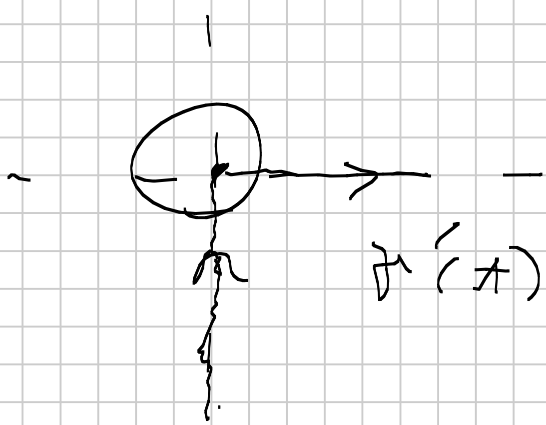
$$x^2 - y^2 = 0 \quad \text{at } (0,0)$$



$$\partial_x F = 2x = 0$$

$$\partial_y F = 2y = 0$$

$$\gamma(t) = \begin{cases} (0, t^3), & t < 0 \\ (t^3, 0), & t \geq 0 \end{cases}$$



$$\gamma'(t) = \begin{cases} (0, 3t^2), & t < 0 \\ (3t^2, 0), & t \geq 0 \end{cases}$$

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deriv vanished

1) Best <u>def.</u>

A set $C \subseteq \mathbb{R}^2$ is called a smooth curve if

1) Connected &

2) Near any pt $(a,b) \in C$, C is either the graph of some C^1 $y = f(x)$ or some C^1 $x = f(y)$

<u>Thm</u> If $F: S \to \mathbb{R}$, S open in $\mathbb{R}^2$, is C^1 & on

$$C = \{(x,y) : F(x,y) = 0\}$$

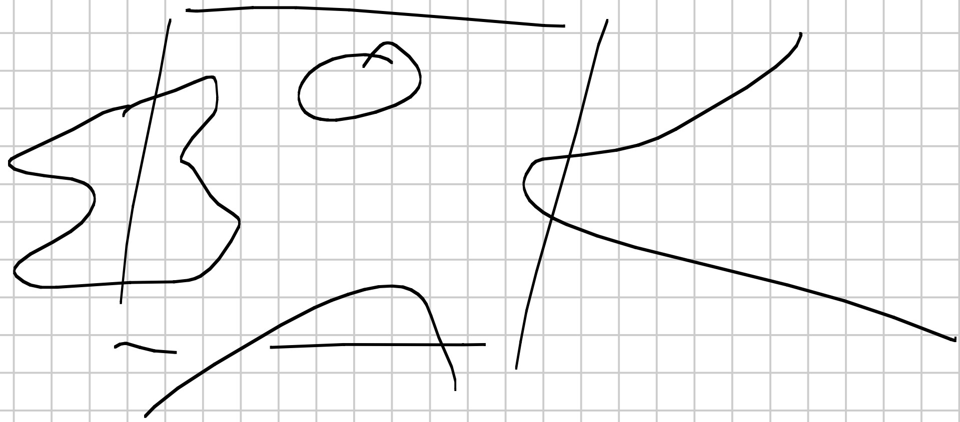
∇F is never 0,

Then

1) If C is connected then C is a smooth curve.

2) If U is open & bounded & $\overline{U} \subseteq S$

then $U \cap C$ is a finite disj. union of smooth curves.



Pf 1) - Impl. fctn thm.

2) $\overline{U}$ - compact.

Say $c_1 \, Q \, c_2$ if whenever S_1 & S_2 disc. C, c_1 & c_2 are in the same set.

Equ. Classes are
Con. Comp. of C.

If c is in some comp. E.

Near c, C is a graph & so is conn

this means all pts of C near c are in the same comp.

take $\overline{U} \cap C$ - closed set.

Compact -
& every $c \in \overline{U} \cap C$ is in an open ball $B_{r_c}(c)$ in one comp.

So $\exists$ finite subcover

$B_{r_{c_1}}(c_1), \dots, B_{r_{c_k}}(c_k)$

cover $\overline{U} \cap C$.
So there are at most k comp.
of C in U.